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Performance investigation between the Newton-Raphson-based optimizer and the recently proposed meta-heuristic algorithms on different theoretical optimization problems

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Abstract

This study presents a detailed performance analysis of a recently proposed meta-heuristic algorithm, the Newton-Raphson-based optimizer (NRBO), across various theoretical optimization problems, compared to other state-of-the-art algorithms. In particular, NRBO is implemented to address four testing problems widely used in the development of most modern meta-heuristic algorithms. Then the results achieved by NRBO are compared with those of two other recently introduced methods, including the Arctic Puffin Optimizer (APO) and the Sharpbelly Fish Optimization (SFO). The comparison results across four criteria indicate that NRBO outperforms the other two in terms of convergence speed, the ability to reach the best fitness values, and, more importantly, the standard deviation (STD). In particular, NRBO consistently outperforms APO and SFO on the Minimum, mean, and maximum fitness values across the four testing problems, with differences in the space solution, variable constraints, and overall complexity. Based on the comparison results and proven capability, NRBO is considered an effective and reliable search method, strongly recommended for use in dealing with such theoretical optimization problems.

Keywords: *Theoretical optimization problem, objective function, benchmark functions, convergence speed Newton-Raphson-based optimizer, Arctic Puffin Optimizer, Sharpbelly Fish Optimization.*

1. Introduction

Light-emitting diode (LED) technology, advanced optimization algorithms, intelligent control systems, and rigorous benchmarking methodologies collectively represent four interconnected pillars driving progress in modern engineering and applied sciences. This paper addresses a problem at the confluence of these domains, drawing on recent advances in each to motivate and contextualize the proposed approach. The following subsections survey the relevant literature across these four thematic groups.

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1.1. LED Technology

LED optical performance is strongly influenced by the physical and chemical properties of luminescent materials embedded within the device. The scattering behavior of ZnS particles at varying sizes was found to significantly affect the optical characteristics—including luminous flux and spectral distribution—of LED systems, highlighting the importance of precise particle size engineering in phosphor-converted LEDs [1]. To address white-balance and color rendering issues, the orange-red emitting $Ba_2Ca(BO_3)_2:Sm^{3+}$ phosphor has been shown to effectively improve chromaticity coordinates and luminance of conventional white LEDs, making it a promising candidate for warm-tone solid-state lighting [2]. Complementarily, the incorporation of SiO_2 particles at different diameters has been demonstrated to enhance LED chromaticity and luminosity simultaneously, offering a practical route to tuning optical quality without major redesign of the LED package [3]. Together, these studies underscore that material-level innovations in phosphor and scattering particle engineering remain an active and fruitful avenue for LED performance optimization.

The development of nature-inspired and population-based metaheuristic algorithms has accelerated significantly in recent years, driven by the need to solve increasingly complex, high-dimensional, and multi-modal optimization problems. A comparative study benchmarked four recently proposed algorithms—Starfish Optimization, Sand Cat Swarm Optimization, Weighted Average Optimization, and Mirage Search Optimization—against global optimization test functions, providing a systematic reference for selecting appropriate solvers [4]. The Elk Herd Optimizer was introduced and applied to minimize electricity production costs in large-scale power systems integrating solar and wind sources, demonstrating strong convergence and competitive performance [5]. The Greylag Goose Optimization algorithm, inspired by the collective migratory behavior of geese, was benchmarked on engineering problems and shown to balance exploration and exploitation effectively [6]. A hierarchical clustering algorithm for multi-modal multi-objective optimization was proposed to preserve solution diversity while maintaining convergence toward optimal fronts [7].

1.2. Optimization Algorithms

For power system economic dispatch—a classical yet persistently challenging optimization problem—numerous metaheuristic solvers have been evaluated. A novel heuristic technique simultaneously handled economic load dispatch (ELD) and economic emission load dispatch (EELD), yielding high-quality solutions at competitive computational cost [8]. The Pied Kingfisher Optimizer was applied to determine the optimal penetration of photovoltaic units in distribution networks [9]. The Growth Optimizer was analyzed on ELD benchmarks with favorable results [10], and the turbulent flow of water optimization demonstrated robust performance on constrained ELD instances [11]. The modified Krill Herd algorithm addressed ELD with valve-point effects and transmission losses [12]. The memetic sine cosine algorithm integrated local and global search to improve ELD scheduling [13], while the Stochastic Shaking Algorithm introduced a novel swarm strategy with verified convergence [14]. The osprey optimization algorithm achieved competitive accuracy on standard ELD test cases [15], and the improved Mayfly optimization algorithm solved combined economic emission dispatch in microgrids with renewable integration [16]. The Five Phases Algorithm, inspired by human social behavior, was applied to ELD with competitive results [17]. The Green ELD problem—incorporating carbon emission penalties—was solved using a novel metaheuristic approach in [18]. Cost-effective dispatch in large-scale systems was achieved with an enhanced manta ray foraging optimizer [19], and the Dandelion optimizer was used for hybrid power plant ELD [20]. Beyond classical evolutionary strategies, quantum annealing was evaluated as a combinatorial optimization tool in a rigorous benchmarking study [21].

Intelligent control of electrical drives and power systems represents a broad and rapidly evolving research area. For induction motor (IM) speed estimation, a sliding mode observer combined with a PID controller was designed and optimized using the Particle Swarm Optimization (PSO) algorithm, achieving improved dynamic performance and robustness [22]. The control of permanent magnet synchronous motors (PMSM) using both direct torque control (DTC) and field-oriented control (FOC) paradigms was comprehensively examined, with integrated control architectures validated on experimental platforms [23]. In photovoltaic systems, a comparative analysis of deep reinforcement learning and conventional maximum power point tracking (MPPT) control under rapidly changing irradiance conditions quantified the trade-offs between adaptability and implementation complexity [24].

1.3. Control Systems

At the distribution network level, the utilization efficiency of electric vehicle (EV) charging stations in commercial grids was improved through optimized management strategies [25]. A war strategy optimization algorithm was applied to simultaneously minimize energy loss and electricity purchase costs in distribution grids by optimally siting clean power sources and soft open point components [26]. Multi-objective dispatch of vehicle-to-grid (V2G)-enabled EVs in renewable-integrated microgrids under diverse load scenarios was formulated and solved in [27]. Economic objective optimization of hybrid systems combining thermal generation, solar PV, and variable-speed pumped storage hydro was reported in [28], while reduction of fuel, energy loss, and investment costs in transmission networks via FACTS devices and renewable integration was demonstrated in [29].

For high-voltage direct current (HVDC) systems, the integration of wind turbines with HVDC transmission was analyzed with respect to power quality and stability [30], and the stability and control of voltage-source converter (VSC)-based HVDC systems were systematically reviewed [31]. A proactive intrusion detection and mitigation framework was proposed for grid-connected photovoltaic inverters to address emerging cybersecurity threats [32]. Battery energy storage systems were shown to enhance HVDC grid stability through coordinated control strategies [33]. A comprehensive market overview of EV batteries provided context for their role in grid-scale energy storage [34]. The mathematical impact of stealthy false data injection attacks on power flow of transmission lines was rigorously verified in [35]. Data-driven coordinated dispatch of source-grid-load-storage systems with renewable integration was proposed using machine learning techniques [36]. Optimal integration of distributed generation was demonstrated to yield technical, economic, and environmental benefits in distribution networks [37]. The sustainability implications of renewable energy penetration on economic dispatch were examined in [38], and a systematic literature review surveyed EV integration in ELD with renewable sources [39].

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Robust dispatch under wind forecast uncertainty for multi-electrolyzer hydrogen production was addressed in [40]. Additionally, the uncertainty quantification of buckling behavior for imperfect structural shells under external pressure was investigated as a complementary control-related structural problem [41].

Benchmarking constitutes a critical methodological foundation for evaluating the validity and comparative performance of optimization and machine learning methods. A seminal analysis identified fundamental flaws in common benchmarking and reporting practices within evolutionary computation, arguing for stricter standards in experimental design [42]. A real-world benchmark problem for global optimization was proposed using practical engineering data, offering a more realistic alternative to purely synthetic test functions [43]. Benchmark experiments for metaheuristic algorithms were accelerated on GPU architectures, dramatically reducing wall-clock time and enabling broader statistical analysis [44]. Challenges and benchmark datasets for machine learning in atmospheric sciences were systematically defined, highlighting the gap between research environments and operational requirements [45]. The case for a standardized software library and enumerated benchmark suite in computer-aided molecular design (CAMD) was articulated to facilitate reproducible research [46].

1.4. Benchmark Functions.

High-fidelity simulation was applied to the Empire microreactor nuclear benchmark problem using Griffin with online cross-section generation, demonstrating the importance of accurate physical models in reference problem solutions [47]. Energy-based physics-informed neural networks were proposed for frictionless contact problems under large deformation and validated against benchmark test cases [48]. The Structural Evaluation Test Unit (SETU) benchmark problem statement was developed to provide a rigorous structural assessment challenge for computational methods [49]. A banking benchmark problem was introduced for teaching model-based systems engineering (MBSE), illustrating the pedagogical value of application-specific benchmarks [50]. InverseBench was proposed as a unified framework for benchmarking plug-and-play diffusion priors across inverse problems in the physical sciences [51]. REALM-Bench extended the benchmark landscape to multi-agent systems operating on real-world dynamic planning and scheduling tasks [52].

In this research, a recently proposed meta-heuristic algorithm called Newton-Raphson-based optimizer (NRBO) [53] is implemented to resolve different theoretical optimization problems. These problems are featured by different main objective functions, dimensions, and variable boundaries. The results achieved by NRBO are compared to other meta-heuristic algorithms, including the Arctic puffin optimizer (APO) [54] and the Sharpbelly Fish Optimization (SFO) [55], to serve the performance analysis purpose.

The main novelites and contributions of this study are as follows:

- Provide a detailed look at the application of a meta-heuristic algorithm to solve the theoretical optimization problem.
- Offer a detailed performance analysis of NRBO compared to the others based on the evaluation of different comparison criteria using the affordable settings in terms of population size and the highest iteration number. Note that these settings are aimed at revealing the difference in the ability to reach the best values of the given optimization problem rather than using the highest settings to determine the best values.
- Demonstrate the effectiveness of the NRBO while dealing with each given optimization problem, which is featured by different characteristics of the main objective function and the other relations, such as dimensions and variable boundaries.

The remainder of this paper is organized as follows: Section 2 formulates the problem; Section 3 presents the proposed methodology; Section 4 reports experimental results; and Section 5 concludes the paper.

2. Problem description

Every optimization problem revolves around two core components: an objective function and a set of constraints. The objective function mathematically models the goal by linking the variables together. Meanwhile, constraints define the boundaries or rules these variables must satisfy. A valid solution must respect all constraints while achieving the best possible objective value.

2.1. The Objective Function

In practice, particularly when applying evolutionary algorithms, the objective is formulated as a objective function to measure solution quality. This function is expressed as:

$$OF(z_1, z_2, \dots, z_r) \tag{1}$$

Where, OF is the fitness value of the considered optimization problem; $z_1, z_2, \dots, z_r$ are the variables that structure the mathematical model of the fitness function; and r denotes the dimensionality of the search space for the given optimization problem.

2.2. The restrictions

Constraints restrict the search space, ensuring that solutions remain realistic and feasible. These restrictions are divided into equality-type and inequality-type restrictions:

- *Equality-type restrictions*

Equality constraints help determine dependent variables during optimization. If we define z_1 as the dependent variable and $z_2, z_3, \dots, z_r$ as the control variables, the equality constraint takes the following form:

$$a_1 z_1 + a_2 z_2 + a_3 z_3 + \cdots - a_r z_r = b$$

where, $a_1, a_2, a_3, \cdots, a_r$ represent the constant parameters determined by the specific problem formulation,; b is a constant values.

- *Inequality-type restrictions*

Inequality constraints define the boundaries of the search space, which meta-heuristic algorithms use to initialize and guide the control variables. They are written as:

$$z_2^{low} \le z_2 \le z_2^{up} \tag{3}$$

$$z_3^{low} \le z_3 \le z_3^{up} \tag{4}$$

$$\vdots$$

$$z_r^{low} \le z_r \le z_r^{up} \tag{5}$$

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In Equations (3) – (5), z_2^{low}, z_3^{low}, …, z_r^{low} and z_2^{up}, z_3^{up}, …, z_r^{up} represent the lower and upper limits for the control variables z_2, z_3, …, z_r, respectively.

2.3. Testing Problems

2.3.1. The first testing problem

Similar to the initial problem, the first optimization problem is also characterized using the objective function and the relevant constraints via specific mathematical expressions as presented below:

$$TP_1(z) = -\sum_{r=1}^{7}\left[\sum_{j=1}^{4}\left(z_j - aM_{r,j}\right)^2 + cM_r\right]^{-1} \quad with\ r = 1,2,\dots,7 \tag{6}$$

where, TP_1 is the value of the first test problem; r is the index of the variable in the summation, ranging from 1 to 7; cM_r is the r-th element of vector cM, which scales the depth of the r-th local minimum; $aM_{r,j}$ represents the position coefficients of the local minima obtained from matrix aM.

Here, aM is a 10×4 parameter matrix containing the coordinates of the target basins:

$$aM = \begin{pmatrix} 4.0 & 4.0 & 4.0 & 4.0 \\ 1.0 & 1.0 & 1.0 & 1.0 \\ 8.0 & 8.0 & 8.0 & 8.0 \\ 6.0 & 6.0 & 6.0 & 6.0 \\ 3.0 & 7.0 & 3.0 & 7.0 \\ 2.0 & 9.0 & 2.0 & 9.0 \\ 5.0 & 5.0 & 3.0 & 3.0 \\ 8.0 & 1.0 & 8.0 & 1.0 \\ 6.0 & 2.0 & 6.0 & 2.0 \\ 7.0 & 3.6 & 7.0 & 3.6 \end{pmatrix}$$

The scaling vector cM is a 1×10 vector:

$$cM = \begin{pmatrix} 0.1 & 0.2 & 0.2 & 0.4 & 0.4 & 0.6 & 0.3 & 0.7 & 0.5 & 0.5 \end{pmatrix}$$

2.3.2. The second testing problem.

Next, we consider TP_2, a classic sphere-like function defined by:

$$TP_2 = \sum_{r=1}^{dim}(z_r - 1)^2 \quad \text{with } r = 1,2,\dots,dim \tag{7}$$

And

In Equations (7) and (8), TP_2 is the value of the second test problem; r is the index of variables required to be determined for obtaining an optimal solution that corresponds to the best value of the primary objective function; dim is the dimension of the considered problem, which is set to 20; z_{TP2}^{low} and z_{TP2}^{up} are the lower and upper limits of the variables.

2.3.3. The third testing problem.

The third benchmark, TP_3 is another variant of the Shekel family, formulated as:

$$TP_3(z) = -\sum_{r=1}^{10}\left[\sum_{j=1}^{4}\left(z_j - a \times M_{r,j}\right)^2 + c \times M_r\right]^{-1} \tag{9}$$

where, TP_3 is the value of the third test problem; r is the index of the variable, ranging from 1 to 10.

2.3.4. Test fourth testing problem

The fourth benchmark is the Rastrigin function (TP_4), which introduces complex local minima:

$$TP_4(z, dim) = \sum_{r=1}^{dim}\left(z_r^2 - 10\cos(2 \times \pi \times z_r)\right) + 10 \times dim \tag{10}$$

In Equation (10), TP_4 is the value of the fourth test problem; dim is the dimension of the search space; r is the index of the variable in the summation, ranging from 1 to dim: $r = 1,2,\dots,dim$.

3. The Newton-Raphson-based optimizer.

Originally, the Newton-Raphson-based optimizer (NRBO) [53] was developed using the Newton-Raphson search approach to solve the engineering optimization problem and was combined with a local optima avoidance mechanism. These are the two foundations of the entire update procedure for the new NRBO solutions.

3.1. The main updated procedure

In this subsection, the original update procedure of NRBO will be briefly described by the following models:

$$S_n^{new} = rn \times (rn \times NV_1 + (1 - rn) \times NV_2) + (1 - rn) \times NV_3 \tag{11}$$

With

$$NV_1 = S_n - \left(rn_1 \times \frac{(S_w - S_{Gb}) \times \Delta V}{2 \times (S_w + S_{Gb} - 2 \times S_w \times S_{Gb})}\right) + \left(\rho_1 \times (S_{Gb} - S_n)\right) + \left(\rho_2 \times (S_{r1} - S_{r2})\right) \tag{12}$$

$$NV_2 = S_{Gb} - \left(rn_1 \times \frac{(S_w - S_{Gb}) \times \Delta V}{2 \times (S_w + S_{Gb} - 2 \times S_w \times S_{Gb})}\right) + \left(\rho_1 \times (S_{Gb} - S_n)\right) + \left(\rho_2 \times (S_{r1} - S_{r2})\right) \tag{13}$$

$$NV_3 = NV - \rho \times (NV_1 - NV_2) \tag{14}$$

In Equations (11) – (14), S_n^{new} is the new newly updated solution n with $n = 1, 2, \dots, N_{ps}$, and N_{ps} is the population size; rn, rn_1, ρ_1, ρ_2, and ρ are the random values in between zero and 1; S_n is the current solution; NV, NV_1, NV_2, and NV_3 are the intermediate vectors that guide the search process; ΔV is the difference of S_w and S_{Gb}; S_w is the worst solution; S_{Gb} is the best solution among the population;

3.1. The local optima avoidance mechanism

In an effort to avoid situations in which the whole search process becomes trapped in local optima that unexpectedly exist in the search space, the authors, in developing NRBO, have proposed the so-called local optima avoidance mechanism (LOA). This mechanism is an additional step

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to refine the newly updated solution after the original update procedure is completed. The mathematical model of this mechanism is presented as follows:

$$S_n^{new} = X_{LOA} \tag{15}$$

With

$$X_{LOA} = \begin{cases} S_n + \vartheta_1 \times (\tau_1 \times S_{Gb} - AF_1 \times S_n) + \vartheta_2 \times \rho \left(AF_1 \times \sum_{n=1}^{Nps} S_n - AF_2 \times S_n \right), if\ AF_1 < 0.6 \\ S_{Gb} + \vartheta_1 \times (\tau_1 \times S_{Gb} - AF_1 \times S_n) + \vartheta_2 \times \rho \left(AF_1 \times \sum_{n=1}^{Nps} S_n - AF_2 \times S_n \right), otherwise \end{cases} \tag{16}$$

In Equation (15) – (16), X_{LOA} is the solutions after refinement using the LOA procedure; ϑ_1 and ϑ_2 are the random values between -1 and 1; τ_1 and τ_2 are the regulating factors; AF_1 and AF_2 are the amplifying terms

4. Results and discussions

In this section, the three algorithms described in Section 3 will be applied to address the four testing problems mentioned in Section 2. The use of those four testing problems aims to provide a detailed look at how effective each algorithm's search capability is in resolving problems with different objective functions, constraints, and boundaries. The performance gap among the three applied algorithms will be revealed through the comparison criteria. However, all three algorithms will be set up to use the same initial control parameters: population size (*Nps*) and the highest iteration index (*HI*). Specifically, these parameters are set to 20 and 100 throughout the four testing problems. These parameters are presented in Table 1 along with the variable limit and definition of the four testing problems. Furthermore, each algorithm will be run for 50 test runs to obtain the best solution before any performance analyses. The results and detailed analyses of the three algorithms' performance will be presented in the next subsections.

Table 1. The specific feature of the four testing problems

Testing problem No.	Dimension	Boundary limits	N_{Ps}	*HI*
1	4	0; 10	20	100
2	30	-100; 100	20	100
3	4	-5; 5	20	100
4	30	-5.12; 5.12	20	100

4.1. The results achieved by the three algorithms on the first testing problem

In this section, the results achieved by APO, NRBO, and SFO will be presented and discussed. Figure 1 shows the results from the three algorithms after 50 test runs when resolving the first test problem described in Section 2. The figure shows that APO outperforms the two remaining algorithms in terms of fitness stability, as evidenced by the smallest fluctuations across all 50 test runs and by the number of times it reaches optimal values, compared to NRBO and SFO. SFO is the worst algorithm in this first testing problem.

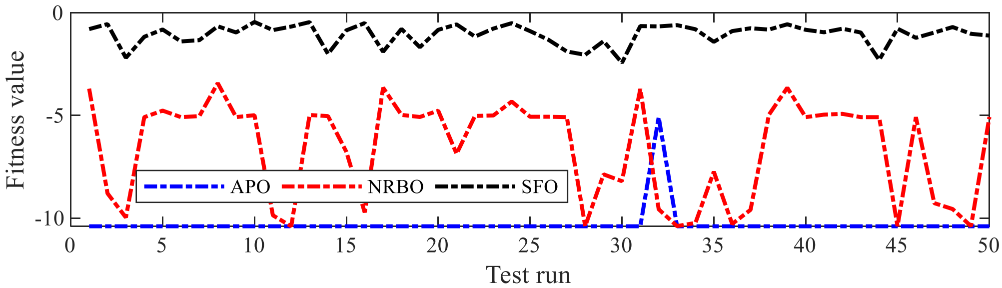

Figure 1. The fitness values obtained by APO, NRBO, and SFO after 50 test runs

while dealing with the first testing problem

Figure 2 shows the three convergences, including a) the minimum, b) the mean, and c) the maximum convergence achieved by the three algorithms while dealing with the first testing problem. In particular, NRBO not only provides faster convergence at the minimum but also clearly performs well in the two remaining convergences, with a clearly faster convergence speed. Furthermore, Figure 3 provides a quantitative comparison of NRBO with SFO and APO across four criteria: minimum fitness (Min), mean fitness (mean), maximum fitness (Max), and standard deviation (Std). The figures indicate that NRBO achieves the same Min values as APO but completely outperforms it in the three remaining criteria.

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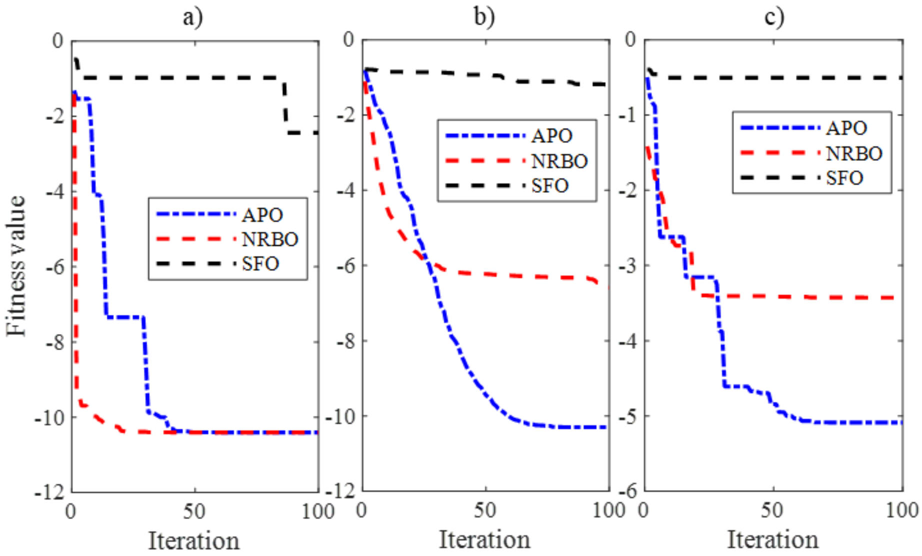

Figure 2. a) the minimum, b) the mean, and c) the maximum convergence achieved by the APO, NRBO, and SFO while dealing with the first testing problem

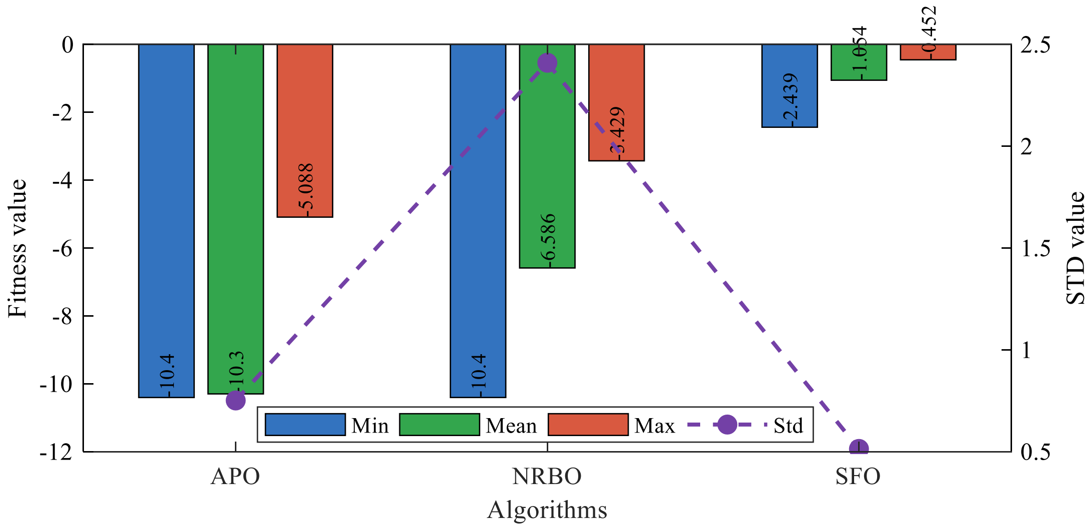

Figure 3. The detailed measurement comparisons of the three algorithms

across different criteria for the first testing problem.

4.2. The results achieved by the three applied algorithms on the second testing problem

This section presents the results obtained by the three algorithms for the second testing problem. Figure 4 shows the fitness value achieved by APO, NRBO, and SFO after 50 test runs. The fitness values shown in the figures indicate that NRBO still maintains the best stability and the shortest time to reach the optimal values of the objective functions, as seen in resolving the first testing problem in the previous section. Besides, SFO is still the worst algorithm in this case, with high fluctuations across all test runs, and it does not even reach the best fitness value across all 50 test runs.

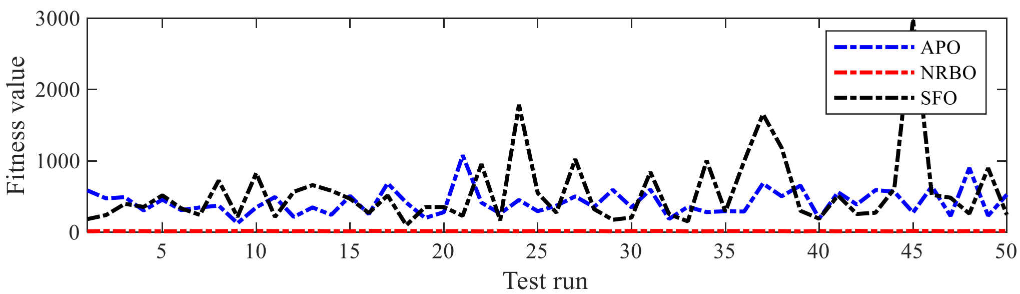

Figure 4. The fitness values obtained by APO, NRBO, and SFO after 50 test runs

while dealing with the second testing problem.

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Figure 5 displays the three convergences achieved by the three algorithms in their best runs. As shown in Figure 4 above, the advantage of the NRBO over the two remaining methods is evident across all three convergences, as reflected in its faster convergence and ability to reach the optimal value of the main objective function. APO and SFO also show improvement as their convergence is observed; however, the performance gap between these two algorithms and NRBO remains significant. Next, Figure 6 provides certain values across four criteria aiming at indicating how well NRBO deals with the second testing problem compared to APO and SFO. In particular, the numerical results in the figure show the wider performance gap between NRBO and APO, which is the second-best method among the three at all four comparison criteria.

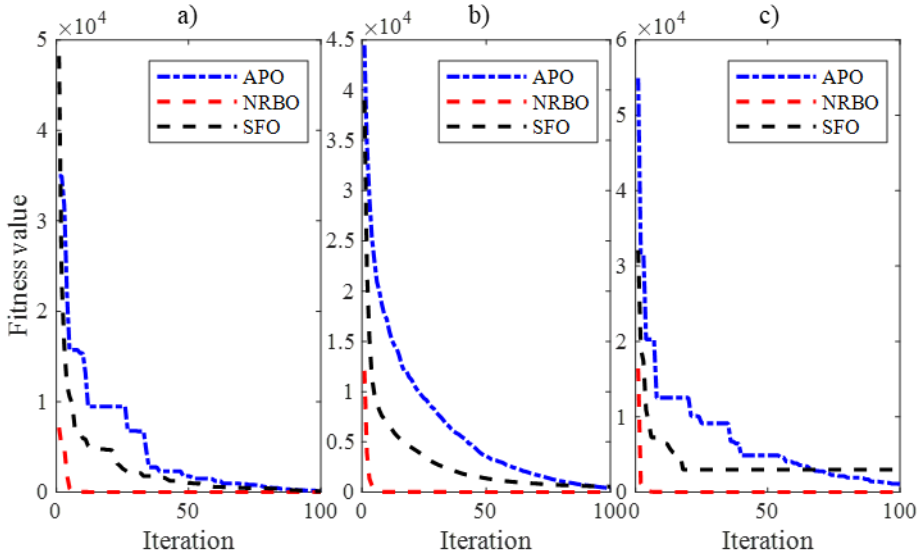

Figure 5. a) the minimum, b) the mean, and c) the maximum convergence achieved by the APO, NRBO, and SFO while dealing with the second testing problem

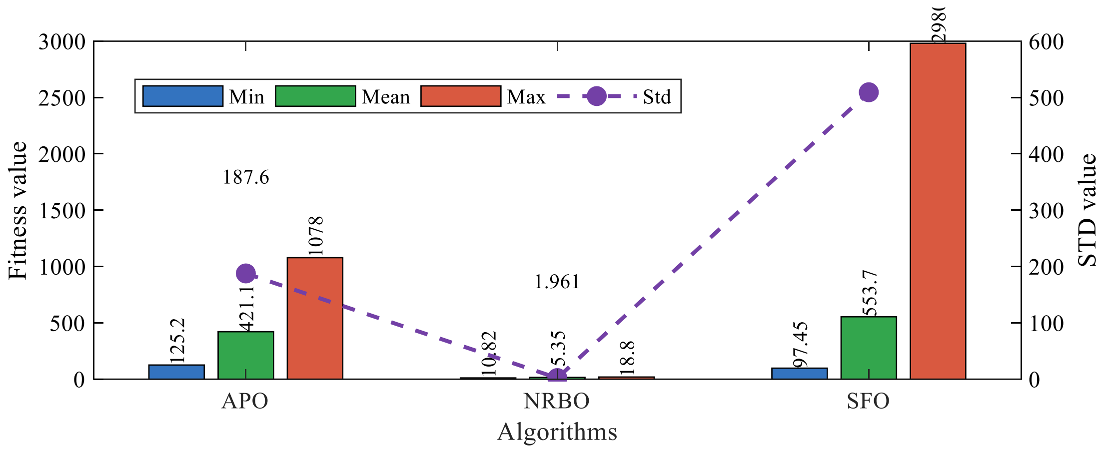

Figure 6. The detailed measurement comparisons of the three algorithms across different criteria for the second testing problem

4.3. The results achieved by the three applied algorithms on the third testing problem

In this section, the results obtained by APO, NBRO, and SFO in addressing the third testing problem will be presented and evaluated. Figure 7 shows the fitness values achieved by the three algorithms after 50 test runs. In the figure, SFO is still the lowest-performing algorithm among the three. APO and NRBO exhibit identical fluctuations across all their test runs, with the number of times reaching the best values of the main objective function being similar between them. Therefore, the observation in Figure 8 will be clearer on the performance difference of these two algorithms. The minimum convergence illustrated in Figure 8a clearly indicates that APO and NRBO can achieve the same value of the main objective function; however, NRBO converges slightly faster after over 50 iterations. Moreover, the advantages of NRBO are more clearly compared to APO and SFO while observing the mean and the maximum convergences in Figures 8a and 8b.

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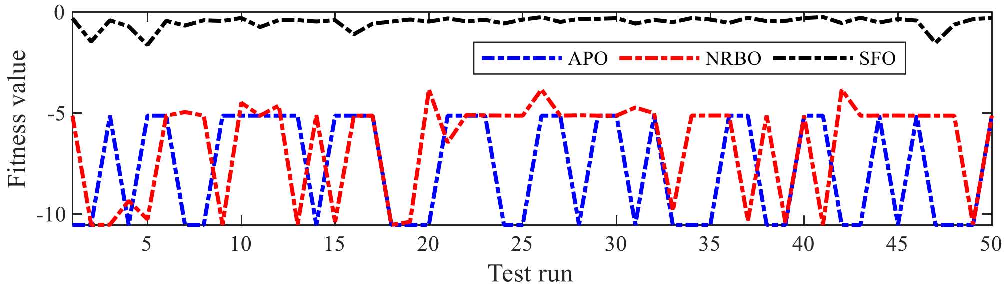

Figure 7. The fitness values obtained by APO, NRBO, and SFO after 50 test runs while dealing with the third testing problem

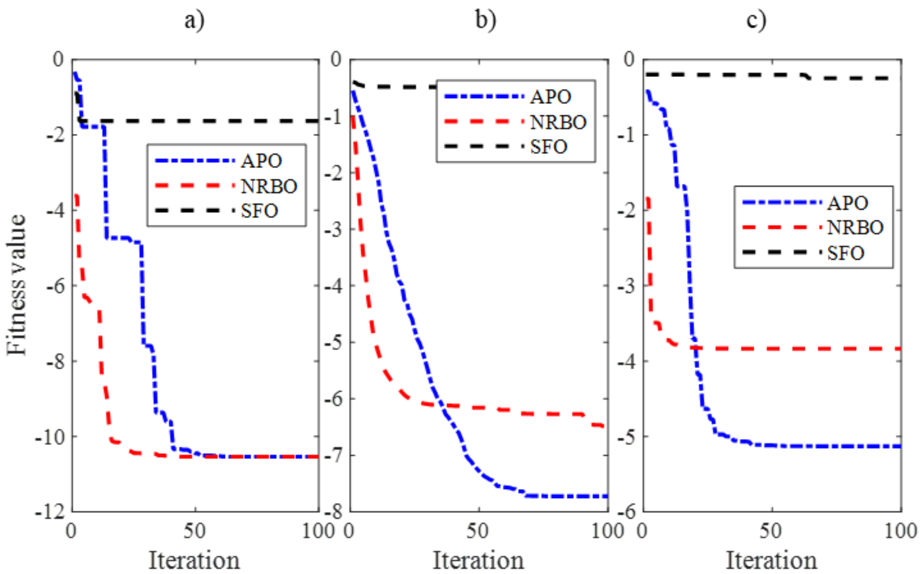

Figure 8. a) the minimum, b) the mean, and c) the maximum convergence achieved by the APO, NRBO, and SFO while dealing with the third testing problem.

Figure 9 provides more details on the actual performance of the three algorithms when resolving the third testing problem, using different comparison criteria and specific measurements. The values and illustration presented in the figure are completely consistent with the previous proofs and analyses. In particular, NRBO achieves the same minimum values as APO and far better than SFO. However, NRBO outperforms APO and SFO in the next three criteria, including the mean, maximum fitness, and STD. STD in this third problem is one of the highlights of SFO compared to the other two; however, due to its poor performance on the first three criteria, SFO is still considered the worst-performing algorithm.

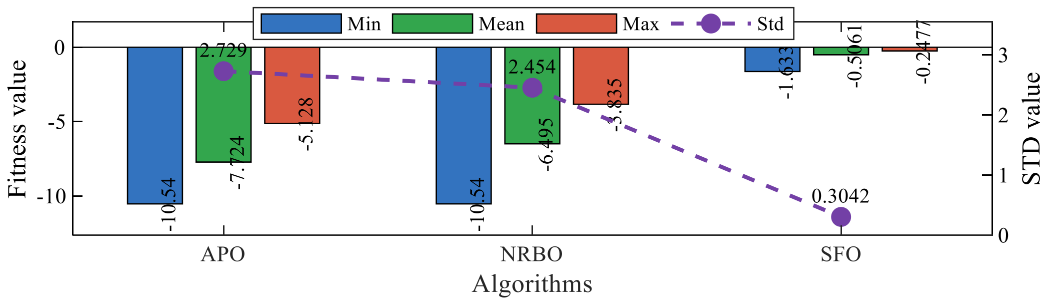

Figure 9. The detailed measurement comparisons of the three algorithms across different criteria for the third testing problem

4.4. The results achieved by the three applied algorithms on the fourth testing problem

In this section, the results obtained by the three algorithms for the fourth testing problem will be presented and discussed. Figure 10 shows the fitness values of APO, NRBO, and SFO after 50 test runs for the fourth testing problem. In the figure, the fitness values obtained by NRBO are nearly flat and located in the lowest positions compared to APO and SFO, with large fluctuations across all test runs. Besides, the flat illustration of all the fitness values obtained by NRBO also indicates that NRBO offers higher stability in search performance than the two remaining methods.

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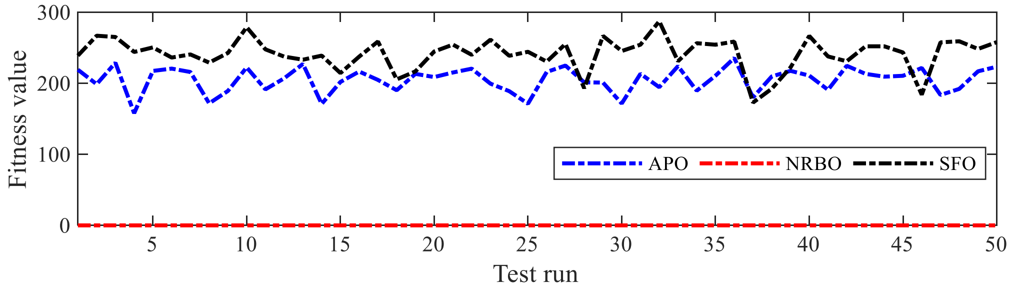

Figure 11 presents the three convergence curves obtained by APO, NRBO, and SFO for the third testing problem. The observation on the three convergence curves clearly indicates that NRBO is far more effective than APO and SFO, both in convergence speed and in achieving the best values of the main objective function in resolving the fourth testing problem. Figure 12 once again consolidates NRBO's superiority over APO and SFO across the four comparison criteria. In particular, NRBO is the only algorithm to achieve absolute zero fitness values and STD, which cannot be reached by either APO or SFO.

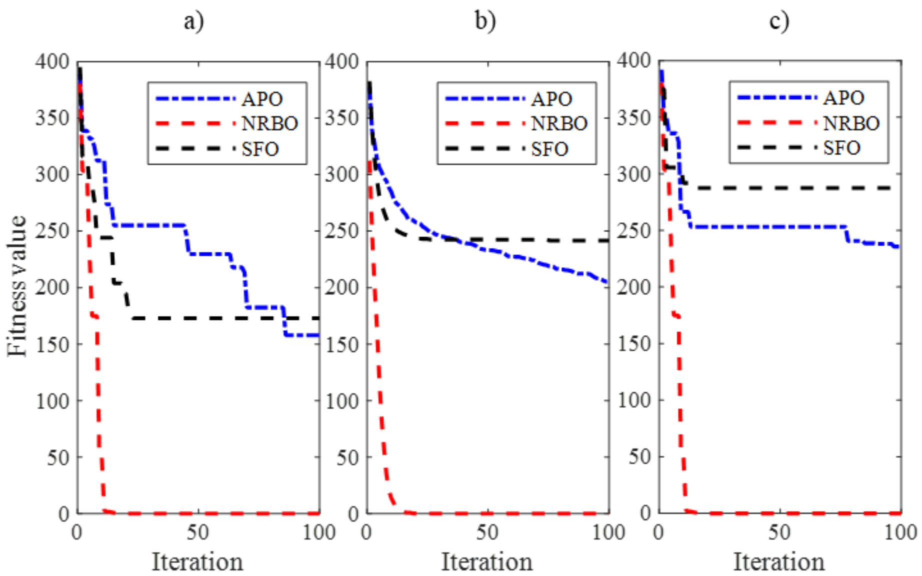

Figure 11. a) the minimum, b) the mean, and c) the maximum convergence achieved by the APO, NRBO, and SFO while dealing with the fourth testing problem.

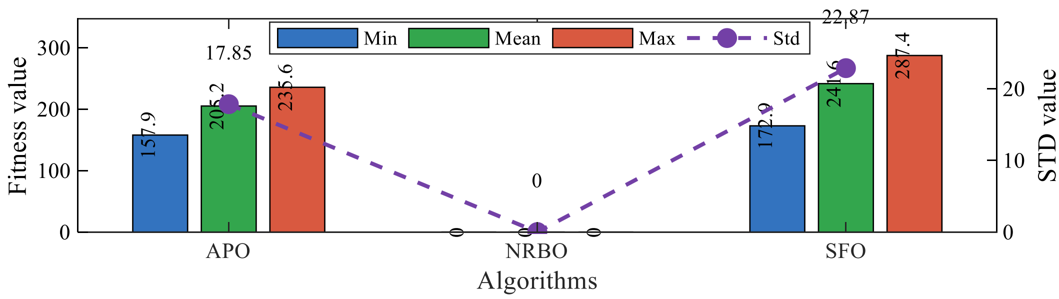

Figure 12. The detailed measurement comparisons of the three algorithms

across different criteria for the fourth testing problem

5. Conclusion

In this study, the performance of the Newton-Raphson-based optimizer (NRBO) is evaluated across different test problems. Besides, the results obtained by NRBO in resolving each testing problem are compared to two other recent proposed meta-heuristic algorithms, including the Arctic puffin optimization (APO) and the Sharpbelly Fish Optimization (SFO). The comparison results clearly indicate that NRBO outperforms APO and SFO across most criteria, including minimum fitness (Min), mean fitness (Mean), and maximum fitness (Max). Regarding the last criterion, standard deviation (STD), NRBO is superior to APO and SFO on three of the four testing problems, except for the third one. In the comparison, the main advantages of NRBO are clearly evident in convergence speed, the ability to achieve the best values of the main objective functions in the given testing problem, and STD values. These are the most important factors that directly affect a particular algorithm's overall capability and effectiveness. Based on all the results and proven capability of NRBO compared to the others, the algorithm is considered to be an effective search method and strongly recommended for use to resolve such problems.

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