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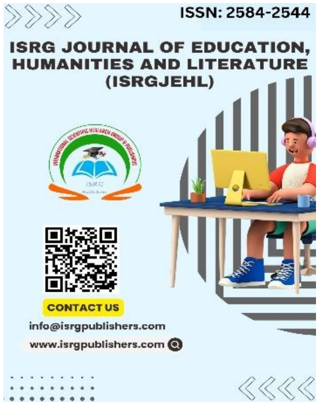

Stages of Critical Thinking in Mathematics

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Abstract

This study aims to develop a grading system for students' mathematical critical thinking based on empirical analysis and synthesis of a theoretical framework of critical thinking. The research uses a qualitative approach with a grounded theory design. The research subjects are students majoring in mathematics education in their 2nd, 4th, and 6th semesters. Data were collected through mathematical problem-solving tests, in-depth interviews, and analysis of student work documents. Data analysis was carried out through the stages of open coding, axial coding, and selective coding. The results of the study produced a model of the progression of mathematical critical thinking consisting of three main domains, namely analysis, evaluation, and synthesis. This model shows that students' critical thinking processes develop gradually, from procedural understanding to reflective and integrative thinking. The findings of this study contribute theoretically to the development of mathematical critical thinking studies and have practical implications for mathematics learning and assessment in higher education.

Keywords: Mathematical Critical Thinking, Ability Grading, Grounded Theory, Mathematics Education

INTRODUCTION

Critical thinking is one of the key competencies of the 21st century that must be developed in higher education (Greene et al., 2012; Davies & Barnett, 2023). Higher education institutions are required to produce graduates who not only master conceptual knowledge but are also capable of analytical, reflective, and evaluative thinking in addressing the complexity of academic and professional problems (Facione, 2015). In the context of mathematics education, critical thinking plays a strategic role because mathematics requires in-depth analytical, evaluative, and logical reasoning skills (Monteleone, Miller, & Warren, 2023). The process of solving mathematical problems involves the ability to understand problems, choose the right strategy, evaluate solutions, and reflect on the thinking process used (Ennis, 2011; Halpern, 2014). However, various studies show that students' mathematical critical thinking skills are still relatively low. Students tend to rely on routine procedures without understanding the meaning of symbols and the relationships between mathematical concepts (Ismaimuza, 2011; Anwar et al., 2020).

The meta-analysis results show that the aspects of evaluation and self-regulation are the components of critical thinking that students

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have the least mastery of (Maharani et al., 2024). This condition indicates that students' critical thinking processes have not developed optimally. One of the main causes is the lack of an operational grading framework to map students' mathematical critical thinking processes (Wang & Abdullah, 2024). Most studies only describe the level of critical thinking ability without systematically examining the stages of thinking development (Mastuti et al., 2022). Critical thinking is an ability that develops gradually and incrementally (Paul & Elder, 2006; Ennis, 2018). Therefore, research is needed that specifically develops a progression of students' mathematical critical thinking based on empirical analysis and the integration of relevant critical thinking theories (Facione, 2020).

LITERATURE REVIEW

Critical thinking is defined as a mental process involving analysis, evaluation, inference, and reflection to determine what to believe or do (Ennis, 1996; Facione, 2011). In mathematics learning, critical thinking is not only oriented towards the final answer, but also towards a deep understanding of concepts and problem-solving processes (Rochmad et al., 2021). Paul and Elder view critical thinking as a process of controlling the quality of thinking by applying intellectual standards to elements of thinking, such as goals, information, assumptions, and implications (Paul & Elder, 2006; Elder & Paul, 1996).

This framework is relevant in mathematics learning because it helps students identify problem structures and evaluate the logic of the solutions used (Nafiah et al., 2024). Ennis categorizes critical thinking into critical thinking dispositions and abilities, which include basic clarification, basic support, inference, advanced clarification, as well as strategies and tactics (Ennis, 2011; Ennis, 2018). In the context of mathematics, these abilities are reflected in how students focus on problems, evaluate mathematical arguments, and choose solution strategies (Ahdhianto, 2020).

Halpern emphasizes that critical thinking involves cognitive skills that can be taught and transferred across contexts, and requires a metacognitive component to monitor the thinking process (Halpern, 2014; Halpern, 2020). This model asserts that mathematical critical thinking skills are not only domain-specific but can also be developed systematically through appropriate learning (Brookhart & McMillan, 2020). Facione, through a Delphi study, identified six core cognitive skills of critical thinking, namely interpretation, analysis, evaluation, inference, explanation, and self-regulation (Facione, 1990; Facione, 2015). This framework is widely used in mathematics education research because it provides clear indicators for assessing students' critical thinking processes (Mastuti et al., 2022). Although various critical thinking frameworks are available, research on the progression of students' mathematical critical thinking processes is still limited and tends to be descriptive (Lestari & Santoso, 2022; Wang & Abdullah, 2024). Therefore, the integration of these various frameworks is necessary to produce a comprehensive and contextual progression model.

RESEARCH METHOD

This study uses a qualitative approach with a grounded theory design because it aims to develop a conceptual model of the stages of mathematical critical thinking based on empirical data (Creswell, 2014; Halpern, 2014). This approach allows researchers to explore students' thinking processes in depth without limiting themselves to a specific theoretical framework from the outset. The research subjects were students in the mathematics education study program in their 2nd, 4th, and 6th semesters who were selected purposively, taking into account variations in mathematical ability and learning experience (Ismaimuza, 2011; Ernawati & Syam, 2021). The research instruments included mathematical problem-solving tests, semi-structured interviews, and analysis of student work documents. Triangulation techniques were used to ensure data validity (Brookhart & McMillan, 2020).

RESEARCH RESULTS

The results of the study indicate that students' mathematical critical thinking processes develop in stages and can be grouped into three main domains, namely analysis, evaluation, and synthesis. The analysis domain is characterized by the ability to interpret and analyze problems, but is still dominated by routine procedures (Facione, 2015; Ennis, 2011). The evaluation domain demonstrates students' ability to assess solution strategies and identify errors, reflecting the development of metacognitive awareness (Halpern, 2020; Rochmad et al., 2021). The synthesis domain is the highest level, where students are able to integrate concepts, make generalizations, and reflect deeply on the thinking process (Paul & Elder, 2006; Facione, 2020).

The progression of students' mathematical critical thinking based on an in-depth analysis of their written work, interview results, and a synthesis of the theoretical framework of critical thinking. The analysis was conducted using a grounded theory approach through the stages of open coding, axial coding, and selective coding until a stable progression model was obtained that was conceptually and empirically validated. The results of the study show that students' mathematical critical thinking processes do not develop linearly, but rather through stages that can be grouped into three main domains, namely: (1) Analysis, (2) Evaluation, and (3) Synthesis. These three domains represent an increase in students' cognitive complexity and metacognitive depth in solving mathematical problems, particularly in the context of linear algebra. In the Analysis Domain: Understanding and Analyzing Mathematical Problems. The analysis domain is the initial level of mathematical critical thinking that is most commonly found in students in their first semester and some students in their middle semesters. In this domain, students demonstrate the ability to understand problems, identify the information provided, and translate questions into mathematical form, but without in-depth evaluation and reflection. Meanwhile, the characteristics of the thinking process in the analysis domain are that students in the analysis domain are generally able to: (1) mention the known and asked data, (2) convert the equation system into symbolic or matrix form, (3) follow routine solution procedures. However, students have not demonstrated the ability to evaluate the strategies used or recheck the correctness of the solution steps. The thinking process is still procedural and mechanistic. These findings are in line with the interpretation and analysis indicators in Facione's framework and basic clarification in Ennis' taxonomy. Findings from Student Written Assignments: Analysis of student written assignments in preliminary studies shows that: Most students are able to begin solving problems, but 83.3% of students make procedural errors in solving two-variable linear equation systems. Errors include algebraic operation errors, incorrect pivot selection, and inconsistencies in elementary row operations. These errors indicate that students have not monitored their thought processes, so they are unaware of the inaccuracy of the steps they have taken. This indicates that the aspect of self-regulation in critical mathematical

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thinking has not yet developed. Meanwhile, findings from student interviews (empirical narratives) based on the interview results revealed that students in this domain (1) followed the solution examples provided by lecturers, (2) considered the answers correct as long as the procedures used appeared to be "according to the formula," and (3) rarely rechecked the results. This narrative indicates that students understand critical thinking as following the correct steps, not as a reflective process. This pattern is in line with the characteristics of unreflective thinkers and challenged thinkers in Paul and Elder's stages of critical thinking. In the Evaluation Domain, which involves Assessing Strategies and Correcting Errors. The evaluation domain shows significant development in mathematical critical thinking processes. Students in this domain not only solve problems, but also begin to assess, compare, and correct the strategies used. The Characteristics of the Thinking Process in the Evaluation Domain show that students in this domain demonstrate the ability to (1) compare more than one solution method, (2) identify errors in the calculation process, (3) correct incorrect steps, and (3) recheck the results with substitutions or alternative approaches. This ability reflects the indicators of evaluation and self-regulation in Facione's framework and advanced clarification and strategies and tactics in Ennis' taxonomy, as formulated in the dissertation theory synthesis. Findings from Student Written Work In student written work in this domain, it was found that (1) students were able to provide corrective notes on incorrect steps, (2) students were aware of inconsistencies in results and attempted to correct them, (3) some students compared the elimination method with substitution. (4) This pattern indicates the presence of early metacognitive awareness, where students begin to control their own thinking processes, although not yet systematically. The interview results show that students in the evaluation domain: (1) Are aware of the possibility of errors even though the procedures have been followed, (2) Recognize the importance of double-checking results, (3) Begin to question whether the methods used are the most efficient. This narrative shows a shift from procedural thinking to reflective thinking, in line with the beginning thinker and practicing thinker stages in Paul Elder's model discussed in the dissertation. The synthesis domain is the highest level in the progression of mathematical critical thinking processes. This domain is only found in a small number of students, especially final semester students with more mature learning experiences. Characteristics of the Thinking Process in the Synthesis Domain Students in this domain demonstrate the ability to: (1) Integrate various mathematical concepts into one solution, (2) Generalize solutions into more general forms, (3) Predict results if parameters are changed. (4) Communicating the thought process logically and reflectively. This ability reflects a high level of synthesis as described in the integration of Facione, Halpern, and Paul Elder's theory in the dissertation. Student work in the synthesis domain shows: (1) Solutions that are not only procedurally correct, but also accompanied by conceptual reasoning, (2) Explanations of why one method was chosen over another, (3) Attempts to generalize solution patterns. Students not only answer "how," but also 'why' and "what if," which are key indicators of advanced critical thinking. In interviews, students in this domain stated that: (1) They are accustomed to double-checking the logic of solutions, (2) The thinking process is considered more important than just the final answer, (3) Mistakes are seen as part of the learning process. This narrative reflects the characteristics of advanced thinkers to master thinkers in Paul and Elder's stages of critical thinking.

Table of Characteristics of Critical Thinking Process Grading in Mathematics

Domain	Process Focus	Key Characteristics	Dominant Indicators
Analysis	Understanding problems	Procedural thinking, following examples	Interpretation, Analysis
Evaluation	Assessing solutions	Detecting errors, comparing strategies	Evaluation, Self-Regulation
Synthesis	Integrating	Generalization, reflection, logical communication	Inference, Explanation

DISCUSSION

The findings of this study reinforce the view that mathematical critical thinking develops gradually and non-linearly (Ennis, 2018; Halpern, 2020). The analysis–evaluation–synthesis progression is consistent with Paul and Elder's stages of critical thinking development, ranging from unreflective thinker to advanced thinker (Paul & Elder, 2006). The integration of the Ennis, Halpern, and Facione frameworks shows that mathematical critical thinking involves not only cognitive abilities, but also metacognition and thinking dispositions (Facione, 2015; Halpern, 2014). These findings enrich the study of mathematical critical thinking, which has previously been more descriptive in nature (Mastuti et al., 2022; Wang & Abdullah, 2024)

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