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CAPABILITY INDICES FOR INDUSTRIAL PRODUCTION PROCESS: A COMPARISON OF CLEMENT'S AND PERCENTILE APPROACHES

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Abstract

The concept of process capability, rooted in statistical process control, emerged as a response to the growing need for systematic approaches to quality management. As industries evolve and consumer demands become more stringent, the need for effective tools to assess and improve process capability becomes increasingly apparent. Hence, this study delves into the context surrounding capability indices and the methodologies employed in evaluating industrial process capability when process data is not normal. In real life, industrial processes are not normally distributed and may contain outliers, thereby; using traditional process capability may leads to wrong conclusions. The aim of this study is to comprehensively evaluate and compare the Clement's, percentile, and Z-score approaches for assessing industrial process capability. Through empirical analysis and theoretical examination, the study would analyze and compare the practical applicability of the Clement's, Percentile, and Z-score approaches using two real-life industrial process data. The empirical evaluation of the methods would be conducted to assess their effectiveness and reliability. The appropriate process capability approach to the two industrial processes would be recommended.

Keywords: Clement method, Non-normal data, Percentile method, Probability Distribution, Process Capability Index.

1. Introduction

In the ever-changing world of industrial processes, efficiency and quality assurance are critical. Capability indices are essential instruments for evaluating the efficacy and potential of industrial processes (Khan et al., 2023). The essence of capability indices lies in their ability to provide quantitative measures of a process's ability to meet specified requirements (Jufri, et, al, 2019). These indices offer insights into process performance, enabling organizations to identify areas for improvement, optimize resources, and ultimately enhance product quality. Process capability based on normal distribution refers to the ability of an industrial process to consistently meet specified quality standards, assuming that the underlying data follows a normal distribution. Capability indices such as Cp and Cpk are sensitive to departures from normality and may provide misleading results if the underlying data distribution is non-normal. Hence, caution must be taken in interpreting capability indices in such cases and there may be need to explore alternative approaches for assessing process performance that are reliable and accurate.

However, achieving and sustaining optimal process capability poses significant challenges, necessitating the utilization of robust analytical tools and methodologies when dealing with non-normal data. Capability indices such as Cp and Cpk are sensitive to departures from normality and may provide misleading results if the underlying data distribution is non-normal. Despite the availability of various capability indices and assessment methods, organizations often face difficulties in selecting the most suitable approach for their specific processes and objectives. This challenge is compounded by factors such as data distribution characteristics, process complexity, and organizational constraints. Without a clear understanding of which method best aligns with their needs and circumstances, organizations may struggle to accurately assess process capability and implement targeted improvement initiatives.

Through the years, various methodologies have been developed to calculate capability indices when applied to non-normal data distributions, each offering unique perspectives and analytical approaches. Ahmad et al. (2008) reviewed the performances of the Clements (1989) non-normal percentile method, the Burr-based percentile method, and the Box-Cox method for non-normal cases. They conducted a simulation study using Weibull, Gamma, and Lognormal distributions to evaluate these methods. Albing (2009) proposed a class of capability indices useful when the quality characteristic of interest follows a skewed, zero-bound distribution with a long tail toward larger values, and an upper specification limit with a pre-specified target value (T = 0). The study focused on process capability indices for the Weibull distribution. Ahmed (2010) discussed the estimation of commonly used PCIs, for nonnormal data using the characteristics of the Weibull distribution. Quantiles were estimated via the probability plotting technique, and control limits were obtained to determine if the process was in statistical control. Percentage points of the fitted distribution were used under the assumption of Weibull distribution. Kantam et al. (2010) studied the point estimates of process capability indices suggested by Clements through simulation when the underlying distribution is the Half Logistic distribution.

Dianda, et al. (2016) conducted a comparative study of several multivariate capability indices, both original and modified, to evaluate their effectiveness in indicating the actual status of a process concerning its specifications. The study considers various scenarios, including different distributions, numbers of variables,

and correlation levels among them. Safdar et al. (2019) proposed a method to estimate four basic indices for non-normal processes using the Johnson system, which comprises three types that translate a continuous non-normal distribution to normal. Taib and Alani (2021) discussed four methods for evaluating non-normal PCIs, including Box-Cox power transformation, weighted variance method, Clements' method, and Darling-Anderson goodness of fit test. The results showed that the process is stable and under statistical control but not capable based on the value of the PCIs, which did not exceed 65%. Wang et al. (2021) proposed modified Clements' PCIs based on a model selection approach, named robust PCIs method, for the location-scale distribution (LSD) family to evaluate the process capability of a production process. Alevizakos (2023) computed the classical indices for discrete data following Poisson, binomial or negative binomial distribution using various transformation techniques.

In this study, we aim to delve into the background and intricacies of capability indices, with a particular focus on the Clement method, and percentile method. By examining their theoretical foundations, practical applications, and comparative advantages, we seek to provide insights that can inform decision-making and drive continuous improvement initiatives in industrial settings.

2. Methodology

The study adopts a comparative research design to analyze and compare the effectiveness of Percentile Method, and Clement's Method. This design facilitates a systematic evaluation of each method's applicability and performance in the context of industrial manufacturing. The data for the study consists of secondary data obtained from two manufacturing industries. Exploratory data analyses were conducted to investigate patterns and check the assumptions underlying Clement's method to determine the appropriate distribution to use. Control charts were plotted for the data, and analyses were performed using R Studio.

2.1 Clement's Percentile PCI Method

Clements (1989) utilized the Pearson curves to provide better estimates of the relevant quantiles. Non-normal Pearsonian distributions include a wide class of populations with non-normal characteristics. This method uses Pearson curves to provide more accurate estimates of $x_{0.00135}$, $x_{0.50}$ (median), and $x_{0.99865}$. Modified Cp and Cpk do not require transformation of the data and they have straightforward meaning which makes them easy to understand.

Clements' estimator for *Cp* and for *Cpk* are respectively

$$C_p = \frac{USL - LSL}{x_{0.99865} - x_{0.00135}} \tag{1}$$

$$C_{pk} = \min\left(\frac{x_{0.5} - LSL}{x_{0.5} - x_{0.00135}}, \frac{USL - x_{0.5}}{x_{0.99865} - x_{0.5}}\right)$$
(2)

$$C_{pl} = \frac{x_{0.5} - LSL}{x_{0.5} - x_{0.00135}} \tag{3}$$

$$C_{pu} = \frac{USL - x_{0.5}}{x_{0.99865} - x_{0.5}} \tag{4}$$

Notably, $x_{0.99865}$ is the 0.99865 quantile, $x_{0.00135}$ is the 0.00135 quantile, and $x_{0.50}$ is the 0.50 quantile calculated with the knowledge of skewness, kurtosis, mean, and variance from the sample data for a non-normal Personian distribution.

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2.2 Percentile-Based Approach

The percentile method replaces the mean in the standard capability formulas with the median of the fitted distribution and the 6σ range of values with the corresponding percentile range.

Then Percentile method capability indices are defined as follows:

$$P_{pk} = \min\left(\frac{P_{0.5} - LSL}{P_{0.5} - P_{0.00135}}, \frac{USL - P_{0.5}}{P_{0.99865} - P_{0.5}}\right)$$
(5)

$$P_{pl} = \frac{P_{0.5} - LSL}{P_{0.5} - P_{0.00135}} \tag{6}$$

$$P_{pu} = \frac{USL - P_{0.5}}{P_{0.99865} - P_{0.5}} \tag{7}$$

$$P_p = \frac{USL - LSL}{P_{0.99865} - P_{0.00135}} \tag{8}$$

$$C_{pm} = \frac{\min\left(\frac{T-LSL}{P_{0.5}-P_{0.00135}}, \frac{USL-T}{P_{0.99865}-P_{0.5}}\right)}{\sqrt{1+\left(\frac{\mu-T}{2}\right)^2}}$$
(9)

where P_{α} is the $\alpha^{th} * 100^{th}$ percentile of the fitted distribution.

3. Results and Discussion

This section presents the results of the analysis conducted on the two datasets used for the research work, the weight of cement and the sugar level used in the production of soft drink. The primary objective is to evaluate process capability indices using the Clement and Percentile methods and to assess the goodness-of-fit for different statistical distributions applied to these datasets.

Truck	Min	Max	Mean	Std error
Truck1	49.15	50.10	49.70	0.0719
Truck2	49.03	49.99	49.51	0.0653
Truck3	49.08	50.00	49.60	0.0594
Truck4	49.07	50.00	49.57	0.0708
Truck5	49.09	50.00	49.56	0.0614
Truck6	49.08	50.05	49.44	0.0679
Truck7	49.03	49.98	49.54	0.0654
Truck8	49.18	49.96	49.60	0.0527
Truck9	49.12	50.00	49.63	0.0660
Truck10	49.92	50.05	49.53	0.0704

Table1 Descriptive Statistics for weight of Cement

The descriptive statistics for the weight of cement across the trucks and processes in Table 1 indicate a high degree of consistency. The weights range from about 49.03 to 50.10, with average weights close to 49.6. For truck 1, the table above shows that the cement weight ranges from 49.15 to 50.10, with an average of 49.70. The standard error is 0.0719, indicating a reasonably precise estimate of the mean. Truck 8 weights range from 49.18 to 49.96, with an average of 49.60. The standard error is 0.0527, indicating high precision. Also, truck 10 weight ranges from 49.92 to 50.05, with an average of 49.53. The standard error is 0.0704, similar to other trucks in terms of precision.

Table 2: Descriptive Statistics for Sugar Level in Coca-Cola Process

Truck	Min	Max	Mean
Process1	35.02	35.32	35.18
Process2	35.01	35.31	35.17
Process3	35.02	35.31	35.18
Process4	35.00	35.31	35.16
Process5	35.00	35.30	35.16
Process6	35.02	35.31	35.19

Table 2 shows that the Sugar level ranges from 35.00 to 35.32, with means around 35.16 to 35.19. Process 1 has a weight range of 35.02 to 35.32 and an average of 35.18, the narrow range shows stable and consistent sugar level control across processes. Also, the Process 5 sugar level weight ranges between 35.00 and 35.30, with an average of 35.16. The range is tight, suggesting consistent measurement. Process 6 sugar level ranges from 35.02 to 35.31, with a mean of 35.19.

The data was tested for normality using the Shapiro-Wilk test. The Shapiro-Wilk test was performed on both data to check if the data is normally distributed.

Table 3 for Cement and Coca-Cola Data Sets

Shapiro- Wilktest	W	P-value	NormalityStatus
Cement	0.96044	0.0000221	NotNormal
CocaCola	0.9987	0.0000001	NotNormal

The result from the Shapiro-Wilk normality test in Table 3 shows that the data for both Cement and Coca-Cola do not follow a normal distribution as shown in figure1 and figure 2. Therefore, neither set of data is normally distributed, which means that normality-based statistical methods may not be suitable without first transforming the data or using non-parametric approaches. To address this, various non-normal distributions such as Weibull, Gamma, Beta, Log-Normal, and Exponential distributions were considered as potential candidates to model the data more accurately. These distributions are often used in industrial and process capability analyses when the data deviates from normality.

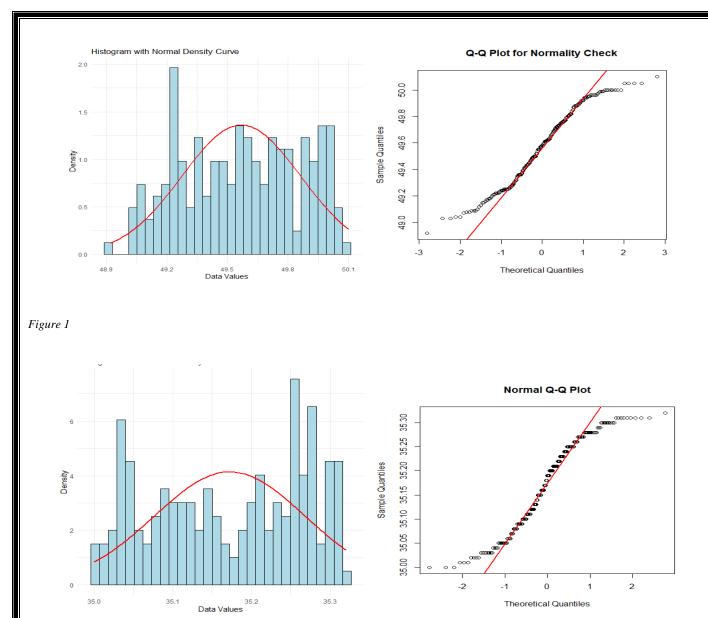


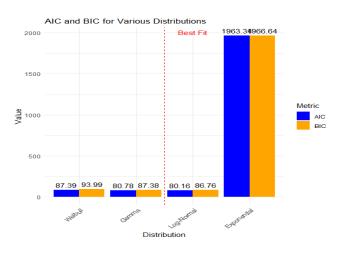
Figure 2

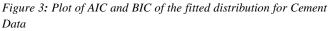
The distributions were fitted to both datasets and the fit of each distribution was evaluated using Log-Likelihood, Akaike Information Criterion (AIC), and Bayesian Information Criterion (BIC). The metrics for each distribution fitted to the dataset are given below.

Table 4 : Distribution Fit Metrics for Cement Dataset

Distribution	Log-Likelihood	AIC	BIC
Weibull	-41.70	87.39	93.99
Gamma	-38.39	80.78	87.38
Log-Normal	-38.08	80.16	86.76
Exponential	-980.67	1963.34	1966.64

The Log-Normal distribution exhibited the lowest AIC (80.16) and BIC (86.76) values, indicating the best fit for cement data. The Exponential distribution, however, was a poor fit with extremely high AIC (1963.34) and BIC (1966.64) values. Figure 3 presents the bar charts of the distribution fit metrics for cement data set.





Weibull 167.55 -331.11 -324.72 Gamma 164.71 -325.42 -319.03 Log-Normal 166.35 -328.71 -322.32 Exponential -820.85 1643.69 1646.89 AIC and BIC for Various Distributions Best Fit Image: Control of the second	Distri	bution	Log-Likelihood	AIC	BIC
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Figure 4: Ploto fAIC and BIC of the fitted distribution for Coca-Cola Data

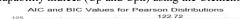
Table 5 and Figure 4 reveal that the Weibull distribution is the best fit for modelling sugar levels in the Coca-Cola process data. It achieved the highest Log-Likelihood (167.55) and the lowest AIC (-331.11) and BIC (-324.72) values, indicating the best balance between model accuracy and complexity. While the Log-Normal distribution also showed strong performance with a high Log-Likelihood (166.35) and relatively low AIC (-328.71) and BIC (-322.32), the Weibull distribution's metrics were slightly superior. The Gamma distribution, with its lower Log-Likelihood (164.71) and higher AIC (-325.42) and BIC (-319.03), proved to be a less suitable fit, while the Exponential distribution demonstrated a poor fit with a very low Log-Likelihood (-820.85) and extremely high AIC (1643.69) and BIC (1646.89) values.

The Weibull distribution, offering the most accurate representation of the data, was selected to derive parameters for the Percentile method in further analysis. Additionally, various Pearson family distributions-Types I, III, VI, and VII-were fitted to both datasets for calculating process capability indices (Cp and Cpk) using Clement's method. Each distribution was evaluated using AIC and BIC values to determine the best fit, ensuring that the most reliable distribution was selected for assessing process capability across both datasets.

Table 6: Pearson family of Distribution Fit Metrics for Cement Dataset

Distribution	AIC	BIC
Pearson Type I	87.79346	104.28505
Pearson Type III	82.06459	91.95954
Pearson Type VI	106.22999	122.72158
Pearson Type VII	104.58992	117.78319

For the Table 6, various Pearson family distributions were fitted and compared using AIC and BIC values to identify the best fit. Pearson Type III shows the lowest AIC (82.06) and BIC (91.96) values, indicating that it is the most suitable distribution for modeling the Cement data. Figure 5 visually supports this conclusion, showing that Pearson Type III consistently has the lowest values compared to the other distributions. Thus, Pearson Type III is identified as the best choice for calculating process capability indices (Cp and Cpk) using the Clement's method.



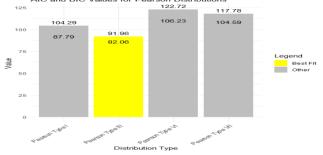


Figure 5: Plot of AIC and BIC of the fitted Pearson Family of distribution for Cement Dataset

Table 7: Pearson family of Distribution Fit Metrics for Cement Dataset

Distribution	AIC	BIC
Pearson Type I	-324.6874	-308.7227
Pearson Type III	-326.7907	-317.2118
Pearson Type VI	-299.7083	-283.7435
Pearson Type VII	-302.9088	-290.1370

From Table 7, Pearson Type III once again provides the best fit, as indicated by its lowest AIC (-326.79) and BIC (-317.21) values. This means that Pearson Type III offers the best balance between model accuracy and simplicity for the Coca-Cola data. Figure 6 reinforces this observation, where Pearson Type III has the lowest plotted AIC and BIC values among the distributions tested.

AIC and BIC Values for Pearson Distribution

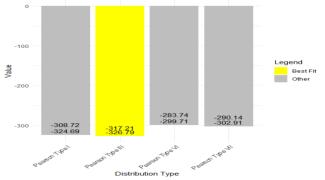


Figure 6: Plot of AIC and BIC of the fitted Pearson Family of distribution for Coca-Cola Dataset.

In contrast, Pearson Types VI and VII have higher AIC and BIC values (AIC: -299.71 and -302.91), meaning they are less suitable for the data. Hence, Pearson Type III is the most appropriate distribution for modeling the data and calculating process capability indices using the percentile method.

3.1 Process Capability Indices

3.1.1Comparison of Process Capability Indices Cement Dataset

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Table 8: Comparisons of Process Capability Indices across Different Methods

СРІ	DIRECT METHOD	LOG NORMAL PERCENTILE METHOD	PEARSONTYPE III CLEMENT'S METHOD
Ср	0.6703	0.6718	0.5794
Cpk	0.6039	0.6009	0.5471
Cpm	0.6574	0.6591	0.5682
Cpmk	0.5922	0.5948	0.5365

Table 8 shows the process capability indices (Cp, Cpk, Cpm, and Cpmk) for three methods: Direct, Log-Normal Percentile, and Pearson Type III Clement Method. The process capability indices (Cp, Cpk, Cpm, and Cpmk) have the following standards: Cp and Cpm values above 1.0 indicate a capable process, with values above 1.33 considered good; Cpk and Cpmk values above 1.0 imply that the process is centered and well-aligned with targets, with higher values indicating better performance. Based on Table 8, the Direct Method (Cp: 0.6703, Cpk: 0.6039, Cpm: 0.6574, Cpmk: 0.5922) and Log Normal Percentile Method (Cp: 0.6718, Cpk: 0.6009, Cpm: 0.6591, Cpmk: 0.5948) both show similar results indicating good process capability and precision; however, they fall below the desired thresholds of 1.0, suggesting inadequate process capability. Also, the Pearson Type III Clement Method (Cp: 0.5794, Cpk: 0.5471, Cpm: 0.5682, Cpmk: 0.5365) consistently yields lower values, indicating poorer performance across all indices. Overall, while the Direct and Log Normal methods provide consistent and reasonable results, all methods indicate that the process does not meet the standards for capability or capable.

3.1.2 Comparison of Process Capability Indices Coca-Cola Dataset

Table 9: Comparison of Process Capability Indices across Different Methods

СРІ	DIRECT METHOD	WEIBULLPERCENT ILEMETHOD	PEARSONT YPEIII CLEMENT'S METHOD
Ср	0.5540	0.4628	0.5367
Cpk	0.5113	0.3734	0.4767
Cpm	0.5495	0.4487	0.5282
Cpmk	0.5071	0.3670	0.4691

Table 9 presents a comparison of process capability indices (CPI) across three different methods: Direct, Weibull Percentile, and Pearson Type III Clement Method. The indices compared are Cp, Cpk, Cpm, and Cpmk, which are used to evaluate the capability of a process to meet specified limits. For Cp, the Direct method (0.5540) and Pearson Type III method (0.5367) demonstrate moderate process potential, while the Weibull Percentile method (0.4628) shows lower capability. In terms of Cpk, the Direct method (0.5113) and Pearson Type III method (0.4767) indicate moderate actual process capability, whereas the Weibull Percentile method (0.3734) reflects lower capability to meet specifications. For Cpm, both the Direct method (0.5495) and Pearson Type III

method (0.5282) show moderate precision, with the Weibull Percentile method (0.4487) indicating slightly lower precision. Finally, for Cpmk, the Direct method (0.5071) and Pearson Type III method (0.4691) indicate moderate capability considering deviation from the target, while the Weibull Percentile method (0.3670) reflects lower capability.

The Pearson Type III methods provide more reliable and consistent estimates of process capability compared to the Weibull Percentile method, which shows consistently lower values across all indices. Given that the data is non-normal, the best method for this dataset is the Pearson Type III method. Although this method does not meet the capability standards (as indicated by its values being below 1.0 for the Cp index), it still represents the best fit for the data because the values are close to the standard.

4. Conclusion

This research evaluated process capability indices for industrial processes using two datasets: cement weight and sugar levels in Coca-Cola production. The study applied Clement and Percentile methods to measure indices such as Cp, Cpk, Cpm, and Cpmk, while normality was tested using the Shapiro-Wilk test, which confirmed non-normal distributions in both datasets. Various probability distributions-Gamma, Log-Normal, Weibull, and Exponential for the Percentile method, and Pearson Type I-IV for Clement's method-were tested for the best fit using Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC).

For the cement dataset, weights ranged from 49.03 kg to 50.10 kg, with a mean of 49.6 kg. The Log-Normal distribution best fit the data using the Percentile method, while Pearson Type III was most suitable for Clement's method. Although none of the methods achieved the ideal process capability index (CPI) value of 1.0, the Log-Normal Percentile method produced values closer to this benchmark, indicating better process precision. In contrast, the Pearson Type III Clement method consistently showed lower index values, suggesting it was less effective in evaluating process capability for cement weight.

In Coca-Cola sugar levels, which ranged from 35.00 mg/L to 35.32 mg/L (mean: 35.16 mg/L), the Weibull distribution provided the best fit for the Percentile method, while Pearson Type III was the best match for Clement's method. The Pearson Type III Clement method yielded more consistent and reliable CPI estimates than the Weibull Percentile method. Despite none of the methods meeting the ideal CPI standard, the study recommends using non-normal distribution-based methods (e.g., Pearson Type III, Weibull, Log-Normal) to improve assessment accuracy for industrial process capability.

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