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Tracking an ESG index to solve a cryptocurrency portfolio

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Abstract

This paper aims to explore the virtual prospects of cryptocurrencies, specifically Bitcoin, Ethereum and others, in order to optimize a green investment portfolio. We include an additional objective, specifically the environmental, social, and governance index (ESG), as a secondary aim. This addition allows investors to define explicit tradeoff thresholds between expected returns and risk, taking the ESG index into account, thereby enabling them to customize their portfolios. Initially, we identify approximate answers to the problem, then, we seek efficient solutions for ESG. Firefly Enhanced Algorithm (EFA) portfolio of cryptocurrencies is solved when combined with Firefly algorithm and tabu search. The findings indicate that the suggested methodology may discern portfolios with favorable returns and risks while accounting for ESG factors.

Keywords: *Cryptocurrency, ESG score, Firefly algorithm, Tabu search, Portfolio optimization*

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I. Introduction

In recent years, the fact that cryptocurrencies and blockchain technology are at the core of the fourth industrial revolution and have the potential to significantly affect a variety of economic and financial sectors has become increasingly apparent [1, 2]. By the end of 2023, the cryptocurrency Bitcoin outperforms all major traditional assets, including stocks, bonds, gold, and oil, in terms of profitability. Despite the challenging macroeconomic conditions and issues inside the cryptocurrency business, its year-over-year growth surpasses 160 percent. Nvidia is the one exception to Bitcoin's underperformance, having surged by 241 percent since the start of the year. The global interest in digital transformation is growing, as various sectors and enterprises see the necessity to rely on digital tools and processes due to new advances and improved technology procedures [3]. Bitcoin offers a decentralized payment method that is not limited by geographical boundaries or the monetary constraints imposed by federal authorities. A recent study suggests that cryptocurrencies like Bitcoin and others are better classified as technology-based products and emergent asset classes than traditional currencies or securities [4].

The potential for portfolio diversification and optimization is drawing a diverse group of investors, individuals, industry participants, and professionals to explore the investment alternatives of this emerging asset class [5, 6, 7, 8].

There are several approaches to solving a portfolio optimization problem, but the most famous one is the Markowitz optimization problem [9]. Nevertheless, if more constraints are introduced, the quadratic method becomes unsuitable for solving the Markowitz issue. In this circumstance, it is advisable to utilize meta-algorithms. Meta-heuristic algorithms refer to algorithms that are commonly inspired by nature and are used to solve nonlinear problems with constraints. The most crucial algorithms include the Genetic Algorithm, Ant Colony Algorithm, Particle Swarm Optimization Algorithm, and Firefly algorithm (FA). Meta-heuristic algorithms, unlike precision-solving techniques, are suitable for tackling large-scale issues and can produce good solutions within a reasonable timeframe. When utilizing precise methodologies or meta-heuristic algorithms to address a problem, it is important to take into account the problem's dimensions and organization [10].

The many hybrid optimizers have undergone significant transformations over the past decade, demonstrating the practicality and effectiveness of utilizing hybridization to develop high-performance optimizers [11]. The Firefly algorithm, introduced by Yang in 2007 [12, 13, 14], is a heuristic optimization technique that utilizes a population-based approach to solve combinatorial and nonlinear optimization problems. The FA is highly efficient in identifying solutions and facilitates straight-forward implementation. Unlike the Particle Swarm Algorithm, the optimization process of FA does not entail looping in a locally optimal solution. Instead, it employs a direct randomized diversification of the search.

A limited selection of results from these experiments indicates the potential use-fulness of Tabu search in many contexts. The inquiry into employee scheduling [15] addressed issues that required solving integer programming problems with formulations using between one and four million variables. It took 22-24 minutes to get solutions that were within 98% of an upper bound on optimality. The study of [16] examined the issue of identifying the coherence of probabilities that indicate whether certain sets of phrases are true. The research also explored the inclusion of probability intervals, conditional probabilities, and minimal changes needed to ensure satisfiability. By combining a Tabu search method with an exact 0-1 nonlinear programming technique to generate columns for a master linear program, we were able to successfully solve a problem with up to fourteen variables. This is a threefold increase in problem size compared to earlier solutions. The quadratic assignment study conducted in reference [17] achieved the most optimal solutions for all evaluated issues from the existing literature, while also needing a shorter amount of CPU time compared to earlier reports. The method also achieved superior solutions com-pared to the best-known solution for a classical benchmark problem [18]. Addition-ally, the method consistently produced solutions of equal or higher quality compared to solutions obtained through simulating annealing, as observed in the maximum satisfiability, graph colouring, and traveling salesman studies [19, 20, 21, 22]. The purpose of tracking is to avoid obstacles smoothly and accurately to obtain sustainable solutions [23]. Based on [24], the ranking of EGS can be performed by considering three types of criteria of different importance. In this way the model becomes more flexible towards certain groups criteria that can be considered differently importance for the final decision rank. To avert crises [25], it is imperative to implement new environmentally friendly changes that will effectively address and overcome them.

II. The Crypto problem formulation

The crypto assets S_1 , S_2, … S_n ($n \geq 2$) with random returns are considered. Let a set of $n \in$ N crypto assets be given. At time $t_0 \in$ R, each asset i has certain characteristics, describing its future payoff: Each asset i has an expected rate of return μ_i per monetary unit, which is paid at time $t_1 \in$ R, $t_1 > t_0$. Let $\mu = [\mu_1, \mu_2, \ldots, \mu_n]^T$. This means if we take a position in $y \in$ R units of asset 1 at time t_0 our expected payoff in t_1 will be $\mu_1\, y$ units. Let σ_i be the standard deviation of the return of asset Si. For $i \neq j$, ρ_{ij} denotes the correlation coefficient of the returns of asset S_i and S_j. The correlation coefficient ρ_{ii} = 1. Let $\zeta = (\sigma_{ij})$ be $n \times n$ symmetric covariance matrix with $\sigma_{ii} = \sigma_i^2$ and $\sigma_{ij} = \rho_{ij}\sigma_i\sigma_j$ for $i \neq j$, and i, $j \in \{1, ..., n\}$. In this notation σ_{ii} is the variance of asset i-th's rate of return and σ_{ij} is the covariance between asset i-th's rate of return and asset j-th's rate of return.

The binary integer programming problem entails the task of minimizing a quadratic objective function while still satisfying linear constraints in the form of equalities and inequalities. In the optimal solution, each variable can be assigned a binary value of either 0 or 1. In scenarios involving multi-criteria optimization, many criteria are simultaneously considered, and it is usually impossible for a single solution to meet all the criteria requirements. It is essential to find a compromise solution that satisfies the decision-preference makers.

A portfolio is defined by a vector $x := (x_1, ..., x_n) \in R^n$, which contains the proportions $x_i \in R$ of the total funds invested in crypto currencies $i \in \{1, ..., n\}$.

We developed the crypto mean-variance optimization model as follows:

min $\text{Xcrypto} = 2^{-1}\text{Xcrypto}^{T}\alpha\ \text{Xcrypto}$ (1)

s. c.

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$$\lambda^{T} X_{crypto} \geq \exp R \qquad (2)$$

$$lb \leq X_{crypto} \geq ub \qquad (3)$$

$$X_{crypto} \geq 0 \qquad (4)$$

$$\sum_{i=1}^{n} (x_i) = 1 \qquad (5)$$

where $\alpha^{T} \in \mathbb{R}^{m \times n}$, $b = \mathbb{R}^{m}$, $\boldsymbol{\lambda} \in \mathbb{R}^{n \times n}$ are given, and $x \in \mathbb{R}^{n}$. Crypto quadratic programming models are a type of nonlinear optimization problem, with some forms being specific instances of linear programming problems.

Quadratic programming components are frequently observed in optimization models. Recall that x is a convex function, which is the objective function (1). Recall that, when $\boldsymbol{\xi}$ is a positive semi-definite matrix, i.e., when $\mathbf{x^t \lambda_y \geq 0}$ for all x. The feasible set is convex because it is a polyhedral set (defined by linear constraint). Consequently, when λ is positive quasi-definite and is positive semi-definite, the Quadratic problem (1) is a convex optimization task. Thus, its globally optimum solutions also happen to be its local optimal ones.

Within the concept of socially responsible investing, a certain subgroup places emphasis on portfolio construction using ESG indices. According to this approach, each company undergoes a comprehensive assessment in the following two three-stage categories: environment, socially and governance [26]. These assessments include key factors such as quality of employment, health and safety, human rights, product stewardship, emissions, board composition and other relevant criteria [27]. The assessment of each company is based on the performance in each category and ultimately leads to the calculation of an overall ESC score for that company. This overall score is the result of the combination of the three-stage score categories and thus provides a valuable sustainability ranking of companies [26, 27].

Based on [28] that maximizing an ESG index is highly desirable, in our opinion the decision maker may not be interested in maximizing the ESG target as it may be a poor performance and high-risk decision. Thus, we propose to first deal with a relaxed version of the classical problem using epsilon dominance [29, 30]. This dominance has been widely used in the literature, for example, to find approximate solutions and later perform post-processing for a specific purpose [31, 32].

III. The experimental model of crypto portfolio

The simulation framework is constructed utilizing the aforementioned mathematical methodology, comprising a total of fourteen cryptocurrencies for the year 2023. The geometric mean, correlation matrix, and covariance matrix were computed using actual historical return data for these cryptocurrencies. Cryptocurrencies were grouped into three sets, based on criteria with similar returns. The proposed approach is tested for its applicability and effectiveness using the actual daily stock closing price time series of cryptocurrencies from January 1, 2023, to December 30, 2023, as referenced in [33] and [34]. ESG data is sourced from Sustainalytics, Inc., located in Boston, MA, USA, and is utilized as an indicator of ESG risk evaluation, with a greater ESG risk score reflecting a less advantageous rating for the organization. These portfolios are formulated as quadratic programming (QP) problems, where the objective is to optimize the portfolio. The optimization is subject to the constraint that the sum of returns for the overall portfolio must equal a specified amount [35].

The covariance matrix was calculated and the Coefficients from it were used for the formulation of the problem. The crypto portfolio is data set as a QP:

$$\lambda^{T} x = 0.347154 x_1 + 2.376808 x_2 + 0.261502 x_3 + 4.025126\, X_4 + 2.053903 X_5 + 0.539857 X_6 + 0.759164\, X_7 + 3.856372 x_8 + 2.683245 x_9 + 5.841272 x_{10} + 7.167325\, X_{11} + 4.053903 X_{12} + 3.539857 X_{13} + 0.759164\, X_{14} + 1.783269\, X_{14} \geq \mathrm{EXP}\ R$$

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12} + x_{13} + x_{14} = 1$$

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12} + x_{13} + x_{14} + x_{14} \geq \mathbf{0}$$

The quadratic problem was solved using Matlab's solver. The Firefly method was adapted to address the portfolio problem outlined in references. The arrangement method was utilized to guarantee that the total weight of all assets in the portfolio is identical to one, instead of perceiving it as a limitation. The problem is resolved thrice using different anticipated rates of return expressed as fractions. The return rates are 0.0050%, 0.0060%, and 0.0070%. Based on the above, we select cryptocurrencies as a secondary objective using the ESG factors, building on the extension of classical optimal solutions. This method is consistent with the a priori approach and allows investors to determine an acceptable level of degradation in the trade-off between classical portfolio objectives (return and risk).

An enhanced firefly algorithm (EFA) for crypto selection

The formulated model (1) - (5) solves a difficult NP-hard issue of nonlinear programming. Conventional robust optimization strategies may not be able to achieve the optimal solution. A novel approach is proposed to efficiently solve the portfolio model by utilizing an improved algorithm that combines the Firefly algorithm and Tabu search for portfolio selection. This approach combines the capability to discover a globally optimal solution (in the case of a multimodal objectives function) with the accurate determination of the optimal solution by reducing the size of the mesh to a predefined tolerance via Tabu search.

Step 1. Determine the EFA parameters: α, β_0 and γ. Set iteration limit – *itlim*. Set diversification limit – *divlim*.

Set iteration counter k=0 and set diversification counter

divcount=0.

Step 2. Initialize fireflies' positions $\{P^k(1),..., P^k(S)\}$, using the three-stage initialization strategy

While (there is improvement of at least one firefly brightness repeat):

Step 3. For each firefly $P^k(i)$ find the brightest firefly it can see.

Step 4. Calculate the new fireflies' positions and update the fireflies' swarm. Update iteration counter: $k = k+1$. Check the stopping criteria and if it is met - go to Step 6.

End While

Step 5. If mod (k/100) = 0, start the *tabu search* procedure.

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Step 6. Show the best obtained solution to the decision maker.

Step 7. Check the stopping criteria. If any of the stopping criteria is met - go to Step 8. Otherwise set a diversification search. Update the diversification counter:

divcont = *divcount* + 1.

Step 8. END.

A diversification strategy is especially applicable in instances when the optimal solutions can only be achieved by overcoming specific obstacles that require making actions with lower evaluations. To determine suitable strategies for overcoming obstacles, a memory function can be developed to categorize the relative desirability of different actions within a specific range.

The concept of "move distance" arises from the observation that certain moves result in more significant alterations to the existing solution compared to others. Within the realm of integer programming, the extent to which a certain action affects the relative feasibility or infeasibility of specific constraints, or modifies the value of certain dependent variables, can serve as the foundation for establishing a measure of distance.

IV. An ESG solved Crypto portfolio

In this part, portfolio optimization is performed using multi-objective meta-heuristic algorithms (firefly algorithms and tabu search). Therefore, in this approach, there is no limit on the objective function X and both forms are considered minimum. The parameter settings for ESG are as follows: x = 20 (population size), $\gamma = 2$, $\beta 0 = 2$, $\alpha = 0.2$, CR=0.2 and F $\in$ [0.2 0.8]. The ESG was run with 20 iterations and 20 populations, and when looking at Figure 1, minimum risk results, ESG found the lowest risk respectively 0.000624538 (Fig. 1.).

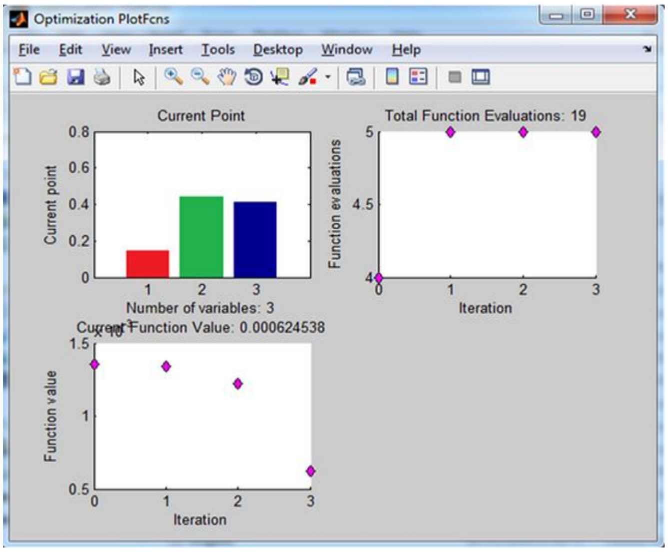

Fig. 1. Optimal crypto portfolio with 0,0050% expected return

The portfolio strategies for risk-minimizing investors for nineteen objective function value calculations and three iterations for the enhanced firefly algorithm are represented in Figure 2. When we searched the expected return of portfolio with 0.0060 %, the ESG found the lowest risk 0.000616333.

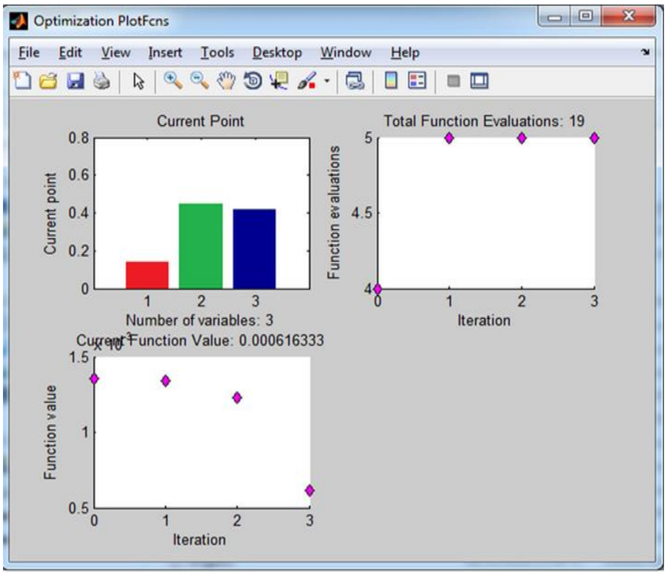

Fig. 2 Optimal crypto portfolio with 0,0060% expected return

The portfolio strategies for risk-minimizing investors for nineteen objective function value calculations and three iterations for the enhanced firefly algorithm are represented in Figure 2. When we searched the expected return of portfolio with 0.0060 %, the ESG found the lowest risk 0.000616333.

TABLE I. THE CRYPTO MODEL WITH AN ESG INDEX

VALUE OF OBJECTIVE FUNCTION	EXP. RETURN [%]	FUNCTION EVALUATIONS	OPTIMAL PORTFOLIO		
			Set 1 [%]	Set 2 [%]	Set 3 [%]
6.6245738266457 12E-4	0.0050	19	5,73	49,03	45,23
6.6163333352404 5E-4	0.0060	19	5,15	49,41	45,42
6.6062962311595 44E-4	0.0070	19	4,54	49,81	45,63

The multi-objective meta-heuristic techniques yielded the following findings for risk minimization: the FA approach achieved the lowest risk value of 0.00604193 with a return of 0.00185234, while the TS method had a higher risk value of 0.007385421 but a higher return of 0.00328731. It exhibited greater efficiency compared to alternative approaches. However, while aiming to maximize the expected return value, the ESG index offers the highest return value of 0.0070 with a comparatively lower risk value of 0.000624538. Our result suggests that EFA with ESG has better performance in optimization through cryptocurrency portfolio because it gives a higher return in size based on 0.0070, with FA showing 0.00185234 and TS 0.00328731. When minimizing the risk of the portfolio, the indicators are FA 0.00604193, TS 0.007385421, and EFA with ESG 0.000624538. This indicate that employing intelligent technologies can provide financial investors with less risk and increased rewards. Hence, this article concludes that multi-objective meta-heuristic algorithms can assist financial investors in selecting the appropriate portfolio by analyzing the outcomes.

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V. Conclusion

Advances in the development of blockchain technology and cryptocurrencies are having a significant impact on supply chain management and the financial sectors - creating more efficient and secure systems for trading digital assets, managing digital identities and implementing decentralized finance (DeFi). The research proposes an improved methodology for globally optimizing green investments in cryptocurrency wallets while taking restrictions into account. It combines two meta-heuristic algorithms, Firefly algorithm and Tabu search. The updated crypto portfolio solution technique achieves an optimal balance between return and diversification.

An enhanced Firefly algorithm was evaluated using a dataset from 2023 that included fourteen actual cryptocurrencies and historical data. The EFA with ESG index, which is experiencing tremendous growth and demonstrating great effectiveness, shows its valuable potential and its core concepts. Possible future effects of this have the potential to create deeper connections between artificial intelligence and mathematical optimization. The approach is convenient for conducting early studies because of its ability to easily launch rudimentary implementations with minimal effort and the ability to expand them if necessary. As more improvements are made, the use of learning methods such as green analysis allows for a more comprehensive use of the two main opposing forces represented by the interplay between restrictions and desired criteria and between strategies of intensification and diversification. These efforts will lead to more effective modifications and provide promising opportunities for further research.

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