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From Continuous Time Double Auctions to Walras Auctioneering: A Broad Dynamic Theory of Demand and Supply

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Abstract

The intersection of the supply and demand curves defines the price of a good in conventional Walrasian auctions. There is a linearly tiny reaction to small perturbations since both curves are generically regular. However, the idea is missing a vital component—transactions themselves. What follows when they take place? We create a dynamic theory of supply and demand based on agents with heterogeneous views to address the topic. The Walrasian mechanism is restored when the inter-auction time is limitless. We confirm empirically on the Bitcoin that when transactions are permitted to occur in continuous time, a unique behavior appears: supply and demand vanish quadratically near to the price. This explains why it is generally noticed that the price impact in financial markets behaves as the square root of the extra volume. The implications are significant because they suggest that prices are extremely sensitive to even slight changes in the market clearing process.

Keywords: *Walras law, Marginal demand and supply, Economic dynamics*

I. Introduction

"Prices are such that supply matches demand" is one of economic science's most overused platitudes. To shed light on how this actually occurs, a Walras auctioneer is typically called upon to measure the supply and demand curves $S(p)$ and $D(p)$, which indicate the entire amount of supply and demand for a particular commodity (or asset) for p.

For the set of preferences corresponding to the current supply and demand curves, the equilibrium price p' is thus such that $D(p') = S(p')$, which maximizes the number of goods exchanged among agents Walras. In actuality, it is difficult to fully understand $S(p)$ and $D(p)$, therefore Walras intended for his well-known tatonnement technique to serve as a tool for observing the supply

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and demand curves. Nevertheless, Walras' approach completely ignores a significant portion of market dynamics.

It explains how a pre-existing supply and demand might lead to a clearing price, but it gives us no information regarding what transpires following the transaction. This makes the Walrasian pricing extremely narrow in scope because the theory vanishes the moment the price is found.

The so-called "order book," as described by Glosten [1], Harris et al. [2], is a workable way to balance supply and demand since it allows each agent to post the amounts they are willing to buy or sell based on a price. The total of all sale (buy) quantities posted at or above (below) price p is thus $S(p)$ (resp. $D(p)$). The auctioneer can then clear the market at each time step by determining the (unique) price such that $D(p') = S(p')$.

Before the development of computerized matching engines, market makers functioned as "active" Walrasian auctioneers in the sense that they added to the order book themselves to ensure steady pricing and orderly trading Glosten and Milgrom [3], Madhavan [4]. This is actually how the majority of financial markets operated.

Even though order book based auctions are getting close to Walras' idealization, they still have a fundamental issue: agents may not always make their intentions clear by placing visible orders out of concern that doing so would reveal information to the rest of the market, among other reasons, Handa and Schwartz [5]. It's possible that only agents who have an immediate need to acquire or sell disclose their plans. The true underlying supply and demand curves, $S(p)$ and $D(p)$ are only expected to be revealed by the order book when they are very close to the transaction price. At that point, however, they become entangled with the orders of market makers and high-frequency traders who engage in strategic "hide and seek" games Bouchard et al. [6]. The visible order book is akin to a Potemkin village, with features that heavily depend on the specifics of the market design (time priority, pro-rata matching, small or large tick, presence of hidden orders, etc. – see e.g. Kockelkoren [7]. It reveals very little about the true underlying supply and demand.

It is consequently challenging to directly observe empirically the dynamics of the entire supply and demand curves $S(p)$ and $D(p)$ (except from niche marketplaces like Bitcoin; refer to Donier and Bouchaud [6] and the section below). However, as supply and demand primarily control price dynamics, we require a workable theoretical framework to account for the (observable) evolution of prices by modeling the (unobservable) evolution of the time-dependent curves $S(p,t)$ and $D(p,t)$, where t is time.

Creating a "Walrasian" explanation of market dynamics would be possible as a result, providing a far greater level of insight than merely speculating on ad hoc stochastic price models like the conventional (geometric) Brownian motion Bachelier [8].

Many concerns, some of which are quite fundamental and practically significant, cannot be answered by these stochastic models and call for an understanding of the underlying dynamics and structure of supply and demand. Price impact Bouchaud [6] is one of them; that is, to what extent does an extra unconditional buy/sell quantity K raise or lower the price? This is crucial for regulators who wish to comprehend market stability and price sensitivity to large "freak" orders (see, for example, Donier and Bouchaud [6] as well as practitioners who wish to calculate the costs related to the impact of their trading strategies Almgren and Chriss [9].

The impact I of a small buy quantity K in a Walrasian context can be demonstrated to be linear in K with ease because the supply and demand curves' slopes around the price p (which would apply in the case of $K = 0$) are generally non-zero.

$S(pK) = D(pK) + K$ is expanded by Taylor expantion about p to the first order in K as follows

$S(p') = (pK - p')\partial_p S(p') = D(p') + (pK - p')\partial_p D(p') + K$……(1)

(1) implies $I(K)$ is equivalent to

$pK - p' = \lambda K$……(2)

$\lambda^{-1} = \partial_p S(p') - \partial_p D(p')$ is greater than zero because s is a strictly increasing function while D is strictly decreasing (see figure 1)

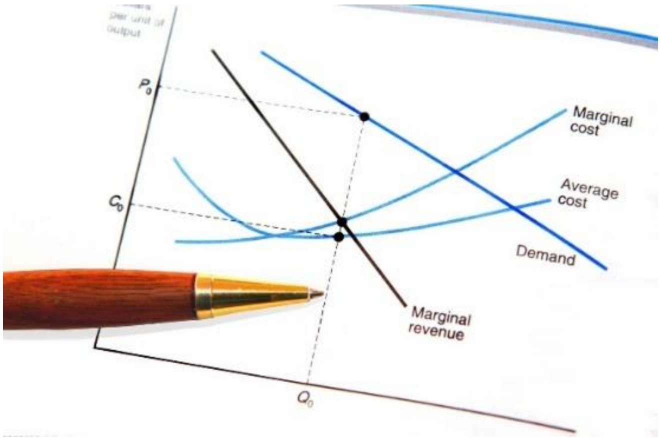

Figure 1: Marginal supply and demand

A linear price response to a perturbation is required whenever the derivatives of the supply and demand curves do not simultaneously vanish at p. Much more complex explanations, such as those offered by the Kyle model Kyle [10], which depicts the interactions between noisy traders, market makers, and knowledgeable traders in the marketplace, can also be used to justify this intuitive conclusion. According to Kyle, the best course of action for market makers is to move the price linearly in the market imbalance, with the market makers' coefficient (Kyles lambda) being inversely proportional to the average volume V traded by the entire market and proportional to the asset's volatility.

The core ideas of market microstructure research have always been price effect, adverse selection, information asymmetry, and market liquidity. The foundational study by Kyle established a foundation in the literature on this topic by connecting all these ideas in a clear and manageable structure. Kyle outlined a game including three different participant types in his model: market makers, noise traders, and insiders.

Numerous studies document a pricing impact that is highly concave, even in the regime with very tiny QV (between, example, 10^{-4} and 10^{-1}). The simple square-root law is given by

$$I(Q) = Y\sigma\sqrt{\frac{Q}{V}}$$

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where Y is a constant of order unity, and it explains a surprising amount of data, regardless of the kind of markets (stocks, futures, FX, options, etc.), geographic regions, historical periods (before to 2005, before the emergence of big HFT, or after 2005), trading methodologies, etc. These days, the empirical evidence is so strong—Donier and Bonart's [11] Bitcoin data, in particular—that it is impossible to avoid searching for a cogent theoretical explanation for a non-linear impact law that is so widespread.

In fact, there are two grounds for thinking that the observed square-root impact cannot be explained by standard equilibrium models. First, in the case of Bitcoin, the square-root impact is precisely obeyed for price changes that are 30 times less than transaction costs and 300 times smaller than daily volatility Donier and Bonart [11]. In this scenario, the difference is around orders of magnitude. It seems like a complete dream to think that the average amateur Bitcoin trader could optimize anything with such accuracy. Secondly, a square-root impact corresponds to

$\partial_Q I(0) = \lambda \to \infty$ where

$\partial_p S(p') = \partial_p D(p') = 0$

Equilibrium models, such as those found in Kyle [10], are unable to replicate this result since they would always predict a linear impact for tiny volumes.

It is more likely that this square-root law is an emergent trait rather than something that market participants voluntarily enforce. Moving away from traditional concepts, Toth et al. [12] presented a thorough scenario for the divergence of Kyles. In that work, the key premise is that order flow and previous transactions themselves interact to form supply and demand curves in significant ways. This is consistent with the theory that universal emergent behavior can be produced by a large population of interacting agents, each of whom uses heuristic decision rules. In actuality, this example highlights the ephemeral features of market dynamics that equilibrium models typically ignore.

II. Analysis of work

There is an extensive body of work on price formation. It is separated into two independent fields, financial economics and microeconomics, which have very different approaches to the issue. The microbusiness community frequently seeks to understand how equilibrium prices are produced in an economy with a predetermined set of actors and preferences and examine their characteristics (distinctiveness, consistency, computability, convergence, etc.). The majority of this view is static.

Financial economists, on the other hand, are primarily concerned with the dynamics of these prices.

On the other hand, the price is forced to have (semi-)martingale characteristics due to the assumption that markets are instantly arbitraged and efficient. This assumption absorbs all knowledge of the true dynamics of supply and demand.

Then, the quantities of interest include the distribution of returns, price volatility, and the microstructural characteristics of the immediate supply and demand as shown in the order book (which, however, only represents a negligible portion of the total supply and demand). This section aims to provide a (rough) synopsis of these two distinct methodologies and situate the current work about them both.

Economics' theory of supply and demand

As previously mentioned, the claim that prices are determined so that supply and demand for every asset in the market equalize has fueled more than a century of economic research on the subject of how prices arise from supply and demand financial system. The first thing to consider is whether this equilibrium is real, distinct, stable understood. In a multi-asset economy, these issues are very subtle, according to Hicks [13].

As many have pointed out, an explanation of the dynamics of prices would be quite important, as such a static account of pricing is not entirely satisfying. Nonetheless, there are a number of different interpretations in the literature regarding the true meaning of dynamics:

1. The way prices converge towards equilibrium might be referred to as dynamics. Some economists have proposed the idea of non-tatonnement, in which agents are permitted to trade before the equilibrium has been reached, to address this issue and the unrealistic fact that the Walrasian mechanism does not allow agents to trade until the equilibrium is reached Fisher [14].
2. It is far from clear if such convergence dynamics, even in the presence of trading, should be identified with the dynamics of market price. Such a model is better understood as a tentative trial-and-error process occurring in functional time [...] [to find] the equilibrium level of prices, rather than as a model of the evolution of a supply-and-demand-driven economy. Thus, we are discussing fleeting dynamics within an otherwise steady universe [15].
3. A multiple period economy introduces dynamics by allowing supply and demand to evolve at each period, resulting in a new price. This weak notion of dynamics is closer to a quasi-static evolution without transactions, where the price is always the outcome of equilibrium supply and demand. However, transactions immediately deplete supply and demand curves, affecting subsequent transactions.
4. Understanding real pricing requires an explanation of supply and demand that is dynamic and ever-changing, particularly in financial markets where transactions constantly disrupt equilibrium and demand and supply interact constantly.

III. Dynamic Theory of Supply and Demand Curves

Definition 3.1: The quantity of supply and demand that would emerge, respectively, if the price were set at p at time t, is represented by the traditional supply and demand curves $S(p,t)$ and $D(p,t)$ (SD).

Definition 3.2: The equilibrium price, p_t' in traditional Walrasian auctions is then set to the value that coincides with both quantities, ensuring that $D(p_t', t) = S(p_t', t)$.

As long as the curves are strictly monotone, this equilibrium is unique. The equilibrium price that is reached as well as the supply and demand curves. Such curves are defined as the derivative of the SD curves

$\rho_s(p,t) = \partial_p S(p,t) \geq 0$

$\rho_D(p,t) = \partial_p D(p,t) \leq 0$

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(i.e $\forall p, \rho S(p,t)$ and $\rho D(p,t)$, representing the quantity of supply and demand that would be metrizable if p is changed to $p+dp$ in the case of supply and to $p-dp$ in the case of demand. (See figure 6)

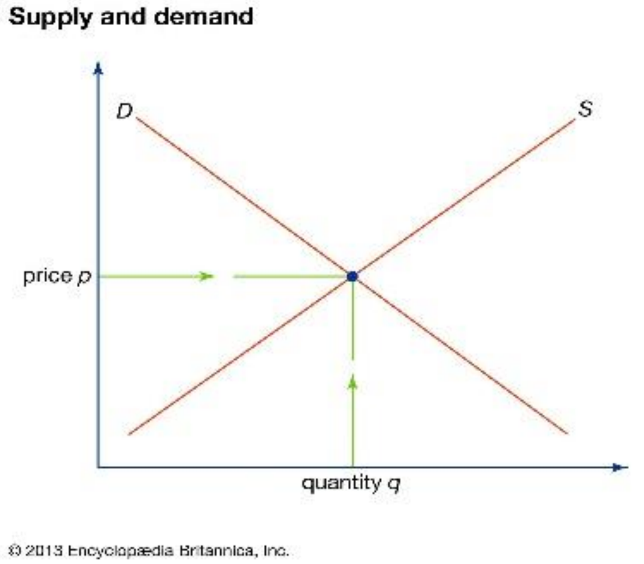

Figure 2

Hence, the density of supply and demand intents around a certain price can be understood as represented by the MSD curves. The MSD curves in Figure 2 (above) are similar to the SD curves: Greater slopes for the SD curves are correlated with higher MSD levels. Supply and demand are preexisting in the Walrasian narrative, and the Walrasian auctioneer searches to find the price point that maximizes the number of possible transactions. Then, at time t, the auction occurs, immediately eliminating all matching orders.

The condition of the MSD immediately following the auction is easy to explain, assuming that all supply and demand intents near the transaction price were disclosed prior to the auction and were matched.

$$\begin{cases} \rho_s(p,t^+) = \rho_s(p,t^-) = 0 & ,p > p_t' \\ \rho_D(p,t^+) = \rho_D(p,t^-) = 0 & ,p < p_t' \end{cases}$$

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