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On Kyburg's assessments of Keynes's graphical, interval valued and lattice representations of probability in Chapter III on page 39 of his A Treatise on Probability

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Abstract

Kyburg's periodic assessments of Keynes's contributions to imprecise probability, interval valued probability, and decision making under uncertainty from 1959 till 2010 suffered from three major problems that Kyburg was never able to overcome in his lifetime. These three problems are the same three problems faced by all members of SIPTA, all papers published in the Journal of Approximate Reasoning that deal with and/or mention the work of JM Keynes since its inception, as well as all heterodox and orthodox economists, especially Post Keynesians like B. Bateman,J. B Davis , T. Lawson ,J. Runde and R. Skidelsky.

The three reasons are

- *A complete and total ignorance of the work of G Boole in his The Laws of Thought (1854) concerning logical probability, interval valued, imprecise probability and the relational, propositional (sentential) logic developed by Boole*
- *A complete and total ignorance of the Boole-Keynes connection. This can only be grasped through a reading of Parts II and III of Keynes's A Treatise on Probability or grasping Keynes's footnote 2 on p.5 of the A Treatise on Probability*
- *A concentration on chapters III, IV and VI of Part I of Keynes's A Treatise on Probability. Kyburg was not nearly as successful as F Y Edgeworth. Edgeworth, but not Kyburg, realized that Keynes's theory was based on lower and upper probability intervals. Edgeworth was able to focus on Keynes's emphasis on the word "between" in chapter III of his A Treatise on Probability in which Keynes spent a great deal of effort discussing his interval valued approach, where Keynes's emphasis and focus was on the phrase" between two numbers (Keynes's italics)." Once this emphasis by Keynes is grasped, then a reader of Keynes's work is in a position to realize that Keynes is using interval valued probability.*

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Despite these deficiencies, Kyburg was able to reject practically all of Ramsey's ludicrous claims that he made about Keynes's logical theory of probability because Kyburg understood Keynes's graphical presentation of interval probability in chapter III on page 39 of the A Treatise on Probability. Thus, although Kyburg's assessment is not of the same quality and depth as Edgeworth's, it is more than sufficient to lead to the rejection of the F P Ramsey critiques in 1922 in Cambridge Magazine and in "Truth and Probability "in 1926.

All past and current academic "interpretations" of Keynes's diagram, with the exception of Kyburg's correct analysis , claim that the diagram on page 39 is a representation of ordinal probability. This makes no sense, as an ordinal approach allows one to conclude only that one probability is either > or < than another probability.

Kyburg presented his lattice structure analysis of Keynes's diagram on p.39 four times -in 1995,1999, 2003, and 2010.All four papers are nearly identical. This paper thus relies on the 2010 paper exclusively.

Key Words: *estimates, non -probabilistic measures, decision weights, non-additivity, imprecise probability, interval valued probability, mathematical lattice structures*

I. Introduction

The paper will be organized in the following fashion. Section Two will deal with Keynes's initial interval valued analysis ,contained in chapter III, which culminated in his rudimentary lattice structure as presented and analyzed on pp.38-40 in the A Treatise on Probability (TP,1921),which Keynes used to provide readers with a rudimentary, basic ,initial, graphical representation of Boole's work, in the form of a type of lattice structure , as later discussed by Keynes in chapters XVI(pp.160-163),XVI,XVII(pp.186-194),XX(pp.233-238) and XXII(pp.254-258) in strictly mathematical terms. Section Three will examine Kyburg's assessment of Keynes's lattice structure on p.39 of the TP. Section Four will examine Kyburg's view that Keynes presented a hint, suggestion, or intuition about interval valued probability, but that Keynes never, ever presented any type of mathematical representation about interval valued probability. Section Five will discuss Kyburg's correct assessment of Ramsey's rejection of such lattice structures /interval valued probability approaches, due to Ramsey's insistence that all probabilities must be linear, additive, exact, precise and unique real numbers, so that mathematical expectation calculations can be performed. This means that it is impossible to arrive at a conclusion that Keynes and Ramsey have a common core that they could agree on, in general.

We can sum this up by emphasizing that the great abyss separating Keynes and Tinbergen is the same as the great abyss separating Keynes and Ramsey. Tinbergen and Ramsey both believed in precise, exact, numerical probability, while Keynes believed in imprecise, inexact, non-numerical (interval) probability.

2. Keynes in chapter III of the TP

The crucial word used by Keynes in chapter III of the TP was the word "between". However, only Edgeworth, alone among academicians in the 20th and 21st centuries ,was able to recognize the implications of that word .No other economist or philosopher grasped the meaning of that word ,a word that shows up again on p.160 of chapter XV following Keynes's extensive use of it in chapter III of the TP, when Keynes introduces the reader to his technical , mathematical analysis of his improved version of Boole's technique, a technique that Boole himself changed based on the improved technique of the mathematician ,Henry Wilbraham, which Wilbraham made in 1854 in a critique of Boole's work.

Keynes's core presentation of interval valued probability is presented on pp.32-36 of the TP after

Keynes made the following observation:

"In fact, underwriters themselves distinguish between risks which are properly insurable, either because their probability can be estimated between comparatively narrow numerical limits (author's emphasis) or because it is possible to make a "book" which covers all possibilities, and other risks which cannot be dealt with in this way and which cannot form the basis of a regular business of insurance, —although an occasional gamble may be indulged in. I believe, therefore, that the practice of underwriters weakens rather than supports the contention that all probabilities can be measured and estimated numerically." (Keynes, 1921, pp. 23-24).

Continuing, Keynes proceeds systematically to present analysis of the use of imprecise probability:

"10. There appear to be four alternatives. Either in some cases there is no probability at all; or probabilities do not all belong to a single set of magnitudes measurable in terms of a common unit; or these measures always exist, but in many cases are, and must remain, unknown; or probabilities do belong to such a set and their measures are capable of being determined by us, although we are not always able so to determine them in practice. 11. Laplace and his followers excluded the first two alternatives. They argued that every conclusion has its place in the numerical range of probabilities from 0 to 1, if only we knew it, and they developed their theory of unknown probabilities. In dealing with this contention, we must be clear as to what we mean by saying that a probability is unknown. Do we mean unknown through lack of skill in arguing from given evidence, or unknown through lack of evidence? The first is alone admissible, for new evidence would give us a new probability, not a fuller knowledge of the old one; we have not discovered the probability of a statement on given evidence, by determining its probability in relation to quite different evidence. We must not allow the theory of unknown probabilities to gain plausibility from the second sense.A relation of probability does not yield us, as a rule, information of much value, *unless it invests the conclusion with a probability which lies between narrow numerical limits* (author's emphasis). In ordinary practice, therefore, we do not always regard ourselves as knowing the probability of a conclusion, unless we can estimate it

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numerically. We are apt, that is to say, to restrict the use of the expression probable to these numerical examples, and to allege in other cases that the probability is unknown. We might say, for example, that we do not know, when we go on a railway journey, the probability of death in a railway accident, unless we are told the statistics of accidents in former years; or that we do not know our chances in a lottery, unless we are told the number of the tickets. But it must be clear upon reflection that if we use the term in this sense, —which is no doubt a perfectly legitimate sense, — we ought to say that in the case of some arguments a relation of probability does not exist, and not that it is unknown. For it is not this probability that we have discovered, when the accession of new evidence makes it possible to frame a numerical estimate. Possibly this theory of unknown probabilities may also gain strength from our practice of estimating arguments, which, as I maintain, have no numerical value, by reference to those that have. We frame two ideal arguments, that is to say, in which the general character of the evidence largely resembles what is actually within our knowledge, but which is so constituted as to yield a numerical value, *and we judge that the probability of the actual argument lies between these two (author's emphasis). Since our standards, therefore, are referred to numerical measures in many cases where actual measurement is impossible, and since the probability lies between (Keynes's emphasis) two numerical measures (author's emphasis*), we come to believe that it must also, if only we knew it, possess such a measure itself." (Keynes,1921, pp.31-32).

Edgeworth reached his conclusions regarding Keynes's interval valued probabilities just from his study of one chapter, chapter III. Edgeworth made it clear to Edwin Wilson that he could not follow Keynes in Part II, which is exactly what Wilson admitted to Edgeworth in private correspondence. Only Borel made his inability to follow Keynes in Part II public in his 1924 review.

In chapter XV, Keynes makes the exact same statement that he made in chapter III:

"The sphere of inexact numerical comparison is not, however, quite so limited. Many probabilities, which are incapable of numerical measurement, can be placed nevertheless between (again, Keynes's emphasis) numerical limits. And by taking particular non-numerical probabilities as standards a great number of comparisons or approximate measurements become possible. If we can place a probability in an order of magnitude with some standard probability, we can obtain its approximate measure by comparison. This method is frequently adopted in common discourse." (Keynes,1921, p. 160).

Keynes 's continuous use of the word" between", to characterize his approach to probability, occurs throughout his analysis on pp.38-40 of his rudimentary, lattice structure that deals with the diagram on page 39(Page 42 of the 1973 CWJMK version of the TP). This diagram has been constantly mistaken for over 100 years as supposedly illustrating ordinal probability. The diagram has nothing to do with ordinal probability at all. All economists and philosophers, who have written on Keynes's chapter III of the TP, have made this error. (See Brady's papers in the references that deal with these erroneous claims- [2021(a, b), 2020(a,b,c, d),2019(a ,b, c ,d),2012.

Keynes then proceeds to work out a total of 17 problems, using upper and lower bounds, in chapters XV (pp.160-163) and XVII (pp.186-194), as well as applying his approximation approach in chapters XX (pp.233-238) and XXII (pp.253-258) of Part III, using his concept of finite probability that is applied to both numerical and non-numerical (interval) probabilities.

Thus, besides the names of Emile Borel, Edwin Wilson, and F.Y. Edgeworth, we can add the names of Issac Levi and Henry E. Kyburg as examples of scholars who were not able to follow Keynes in Part II of the TP. Borel and Wilson, some 15 and 11 years, respectively, after their first reviews in 1924 and 1923, were eventually able to grasp what Keynes (and Boole) were doing.

3. Kyburg on Keynes's lattice structure

Kyburg, despite his other deficiencies with respect to Keynes's mathematical demonstrations concerning his non numerical (interval) probabilities, which are based on Boole, is the only academic who, based on his study of the diagram on p.39, correctly realized that Keynes 's diagram is a version of a mathematical lattice structure:

"This argument suggests, as does the illustration in Keynes (TP, p .39) … that probabilities form a lattice structure…Upper and lower bounds for any probabilities exist, of course, --namely ,0 and 1--- but the question is whether the meet and join of any two probabilities exist. A definitive answer is hard to come by for Keynes…. Nevertheless, the list of properties on page 41 of TP suggests that the answer is affirmative. Probabilities lie on *paths*, each of which runs from 0 to 1." (Kyburg,2011, p.25).

[author's note-this is not the case for the interval valued probability, OVA, which is the g l b and lies between 0 and .5 .Kyburg was unable to realize that the draftsman ,who provided the diagram to Keynes for publication in the TP, made three errors- first ,OVA was supposed to have been drawn intermediate between 0 and 1;in fact, it is drawn 2/3 of the way between 0 and 1.Given that OVA is symmetric, the estimated value of V ,which lies symmetrically in OA ,is .25 [0, .25, .50) and not [0,.335 ,.67].This makes it clear that Keynes did not think it possible "… that there should be an unbounded sequence of ever greater lower bounds…though it is of course mathematically possible."(Kyburg,2011,p.25]; second, the ZWY path was incorrectly drawn(all of the paths represent second order quadratic equations ; and third , U, on the OUI path, should be drawn at the vertex of OUI, not off to the right of the vertex.]

Keynes's extensive footnote two on p.161 makes it clear that Keynes is providing in his diagram on p.39 a representation of Boole's technical analysis ,which involves solving for the roots of second and third order equations (inequations) ,which would require nonlinear paths that are non-additive .The linear and additive path is 0AI,which would require linearity and additivity .This represents Ramsey's view of probability .It is thus easy to conclude from this diagram that Ramsey's theory can only be a very special case of Keynes's far ,far more general theory.

Kyburg also disposed of Ramsey's incoherent claims concerning Keynes's degrees of probability and degrees of belief. Ramsey made the following truly bewildering claim:

"But if, as Mr. Keynes holds, these things are not always expressible by numbers, then we cannot give his statement that the degree of the one is the same as the degree of the other such a simple interpretation, but must suppose him to mean only that there is a one-one correspondence between probability relations and the degrees of belief which [p.161] they justify.

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This correspondence must clearly preserve the relations of greater and less, and so make the manifold of probability relations and that of degrees of belief similar in Mr Russell's sense. I think it is a pity that Mr Keynes did not see this clearly, because the exactitude of this correspondence would have provided quite as worthy material skepticism as did the numerical measurement of probability relations. Indeed, some of his arguments against their numerical measurement appear to apply quite equally well against their exact correspondence with degrees of belief; for instance, he argues that if rates of insurance correspond to subjective, i.e. actual, degrees of belief, these are not rationally determined, and we cannot infer that probability relations can be similarly measured. It might be argued that the true conclusion in such a case was not that, as Mr Keynes thinks, to the non-numerical probability relation corresponds a non-numerical degree of rational belief, but that degrees of belief, which were always numerical, did not correspond one to one with the probability relations justifying them." (Ramsey,1926, Truth and Probability. In Kyburg and Smokler, (eds.),1980, p.27).

Kyburg's assessment of Ramsey claims is very critical:

"This is an odd thing to say, since it is apparently Keynes's intuitions about rational belief that lead him to this view about probability, rather than vice versa .In any event ,since the structure of this manifold of probabilities is very different from the structure of the reals between 0 and 1,to which Ramsey wished to reduce all degrees of belief and all probabilities, it is a pity that Ramsey did not provide more motivation for his drastic reduction of Keynes's rich manifold of probabilities[author's note -Kyburg is referring to Keynes's mathematical, lattice structure in the diagram on page 39, with its glb and lub of the TP ,as opposed to Ramsey's "structure", which is the horizontal, linear line 0AI]to the simple(alleged)structure of degrees of belief."(Kyburg,2010,p.27)

Kyburg's point is that Ramsey's structure can't be represented as a mathematical lattice structure unless one wants to view the single, horizontal, linear line, OAI, as a degenerate lattice structure.

It is clear, then, that only in the special Keynes case where V(a/h) =w and w=1,where $0 \leq w \leq 1$,so that all probabilities become linear and additive since the lower probability bound =the upper probability bound, that Ramsey's theory becomes a very special case of Keynes's general theory.

4. Kyburg on Keynes's interval valued approach

Kyburg next proceeds to examine Keynes's probability intervals from a graphical perspective only, based on Keynes's initial discussions of interval probability in chapter III of his TP:

"The set of sub intervals of (0,1)does form a lattice under the natural ordering (p ,q)(r ,s) if and only if every point in (p ,q) is less than any point in (r,s).The meet of (p, q) and (r, s) is just the degenerate interval (min{r, p},min{r, p})and similarly for the join. On this interpretation of the values of probability we have no difficulty in accommodating Keynes's graph." (Kyburg,2011, p.26)

Kyburg is correct in his analysis. Thus, Kyburg gives Keynes credit for being able to construct a purely graphical-diagrammatical, but not mathematical, approach to interval probability. Keynes's graph was the foundation for Kyburg's assessment that Keynes had made some hints, suggestions, ideas, intuitions, etc., about interval probability (See Kyburg (1999); however, Kyburg insisted that Keynes had never, ever developed a mathematical analysis of interval probability). Unfortunately, Kyburg never, ever read Part II of the TP, where Keynes developed just such an advanced, mathematical approach to interval probability, based on Boole's earlier work. Kyburg's repeated insistence , over some 50 years, that Keynes did NOT develop a mathematical theory of interval valued probability, explains, to a large degree, the ignorance of SIPTA members concerning Keynes's accomplishment, an accomplishment that served as the foundations for Volume I of Keynes's 1930 *A Treatise on Money*, Keynes's 1936 *General Theory*, as well as Keynes's 1938-1940 exchanges with Tinbergen over the merits of precise versus imprecise applications of probability in macro econometrics modeling of investment over the business cycle. The complete lack of understanding of what it was that Keynes had accomplished in 1921 is still the case today in 2024.

However, Keynes's graphical construction did give Kyburg the tools he needed to easily refute Frank Ramsey's attacks on Keynes and logical probability, with a few, minor exceptions. We will do this in Section Five below.

5. Kyburg 's refutation of Ramsey's attacks on Keynes and logical probability

Consider Kyburg's statement below:

"What is curious is that the mathematician -philosopher, Frank Ramsey, paid no attention to this structure in his review of the Treatise (Ramsey, 1922), though he did attack the claims that some probabilities are incomparable and that some were non-numerical...In any event, since the structure of this manifold of probabilities is very different from the structure of the reals (author's note -real numbers), to which Ramsey wished to reduce all degrees of belief and all probabilities."(Kyburg, 2010, p.26).

In fact, Ramsey *deliberately* ignored Keynes's graphical analysis because Ramsey could easily see from the diagram that his theory was represented completely by the linear OAI line segment, which makes Ramsey's theory only a very special case of Keynes's general theory.

Kyburg's examination of Keynes's logical, objective relation of probability suffers from his having overlooked the extensive Boole-Keynes connection. See Kyburg, 2010, pp.23-27,31-32).

Keynes's

- logical relation of objective probability is the same as Boole's
- relational, propositional, mathematical, formal, symbolic logic is the same as Boole's
- Interval valued, non-numerical probabilities are the same as Boole's
- Keynes's logical theory of probability is based on Boole's logical theory of probability
- Keynes's critique of the POI is based on Boole's critique of the POI

Kyburg conceded a point to Ramsey that he would not have made if he had realized that Ramsey's entire critique of Keynes's views between 1922 and 1926 was based on one enormous, gargantuan, basic, foundational error. That error is to have substituted Moore's Platonic, metaphysical, intuitive, logical relations about "the Good "for Keynes's Boolean, objective, logical relations about "the

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Probable." In other words, Ramsey drew a completely false analogy between Moore's Platonic "Intuitionism "and Boole's requirement that a decision maker can intuit a logical connection between sets of RELATED propositions. All of Ramsey's supposed examples of the application of Keynes's supposedly, erroneous propositional logic involve UNRELATED propositions. All orthodox and heterodox economists, writing either on the TP or the connections between the TP and GT for the last 100 plus years, make the same, identical error. The best example of this error is Lang (1964, pp.295-302). There is a direct one-to-one onto correspondence between Lang's many errors made about Keynes and Moore's Intuitionism and the work of Bradley Bateman, John B. Davis, Tony Lawson, Robert Skidelsky, Jochen Runde, Donald Gillies, Darrell Rowbottom, etc. Interestingly, Lang's article has never been cited by any economist or philosopher doing work on Keynes of the connections between Keynes's TP and GT.

Kyburg conceded the following:

"It must be admitted …that Keynes did waffle on the objectivity of the probability relation." (Kyburg,2010, p.27).

Kyburg overlooks that Keynes is not talking about Platonic relations, as insinuated by Ramsey in his claim that he could not perceive any such logical relation that" …relates any two given propositions." (Kyburg,2010, p.27),

The author has written some 100 papers dealing with Ramsey's catastrophic blunder about

"…relates any two given propositions."

Keynes's Boolean theory *relates some sets of related propositions. Unrelated propositions are excluded from both Boole's and Keynes's logical theories.*

Kyburg does realize where the suppositions of Ramsey lead to:

"Now it is all very well for Ramsey modestly to admit that he sees no logical relation of probability such as the one Keynes seeks to draw our attention to, but it is clear that Ramsey wants to go further than that. Ramsey wants to claim that there is no such relation." (Kyburg,2011, p.29).

Note that neither Keynes nor Russell could have simply stood up at an Apostles meeting and stated that the logical relations being used in Keynes's logical theory of probability in his 1921 book are Boolean, not Platonic, as this would have led to the immediate end of Ramsey's career, as well as that of R B Braithwaite's career, both of whom had become members of the Apostles in 1921.

Kyburg was completely unaware of the foundation in Boolean logic that permeates Keynes's TP. However, he was able to see what Keynes was driving at on pp.38-40 of the TP, combined with the diagram on p.39, which is an early version of a mathematical lattice structure developed by Keynes to demonstrate Boole's indeterminate and imprecise interval valued probabilities .Unfortunately, only Kyburg was able to correctly analyze Keynes's graph on p.39 of the TP among economists and philosophers in the 20th and 21st centuries.

6. Different proofs showing that Ramsey's theory of probability is a very special case of Keynes's general theory of probability

There are three different ways, all very simple and direct, to show that Ramsey's theory is a very, special case of Keynes's general theory.

The first way is simply to visually inspect Keynes's diagram on page 39. It is obvious that Ramsey's theory is the linear, additive line OAI, where all probabilities are precise and exact numbers. All the other parts of Keynes's diagram represent nonlinear, non-additive and imprecise interval probabilities that form a lattice structure.

The second way is to realize what it means in Keynes's theory when he measures the evidential weight of the argument, V, by w, so that V(a/h) = w, where $0 \leq w \leq 1$, and w = K/(K+I) ,where K = absolute amount of Knowledge and I = absolute amount of Ignorance. Ramsey's theory is the claim that

w =1 at all times.

Keynes's theory is the claim that

$w \leq 1$.

Ramsey's theory, the claim that

w=1 at all times,

is a special case of Keynes's theory,

$w \leq 1$.

The third way of proving that Ramsey's theory is only a very special case of Keynes's theory is to apply Keynes's conventional coefficient of weight and risk, c. Keynes's conventional coefficient, c, has been rejected by all heterodox and orthodox economists who have done work either on the TP or the connections between the TP and GT (See, for example, Basili and Zappia,2009,2021).

The Ramsey approach is the mathematical translation of Jeremy Benthem's Benthamite Utilitarian approach. Bentham's approach was that the whole can't be anything more than the sum of the individual, atomic parts. However, this requires the assumptions of additivity and linearity. Bentham assumed also that all decision makers can calculate the odds exactly. Keynes showed that this was not the case. Keynes's demonstration, taken from chapter 26 of his A Treatise on Probability (1921; TP), of the special case nature of any expected value(utility) approach, based on the purely mathematical laws of the probability calculus, which are used to support the application of mathematical expectations, shows this to be a very special case where w=1. Bentham claimed that all individuals have the capability to calculate the odds and outcomes and act on the mathematical expectation or the expected utility (the probability times the utility of the outcome) in a rational way. This can be expressed by the following, where p is the probability of success, q is the probability of failure, and A is the outcome:

Maximize p A.

Ramsey's version of this is to Maximize pU(A), where p is a subjective probability that is additive, linear, precise, and exact and U(A) is a Utility function. The goal is to

Maximize p U(A).

Keynes rejected Ramsey's theory except as a very special case that would only hold under the special assumptions of Ramsey's subjectivist, Bayesian model-that all probabilities were additive,

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linear, precise, single number answers that obeyed the purely mathematical laws of the probability calculus.

Keynes specifies his conventional coefficient of risk and weight, c, model in chapter 26 of the TP on p.315 and footnote 2 on p.315, as a counterweight to the Benthamite Utilitarian approach of Ramsey. He extends it to include U(A), the utility of an outcome, as opposed to A, an outcome, on p.320 of the TP:

"In conclusion we may discuss a little further the conception of 'moral' risk, raised in § 8 and at the end of § 9. Bernoulli's formula crystallizes the undoubted truth that the value of a sum of money to a man varies according to the amount he already possesses. But does the value of an amount of goodness also vary in this way? May it not be true that the addition of a given good to a man who already enjoys much good is less good than its bestowal on a man who has little. If this is the case; it follows that a smaller but relatively certain good is better than a greater but proportionately more uncertain good." (Keynes,1921, p.320) Essentially, Keynes's generalized model is given by c=2pw/(1+q) (1+w), which is multiplied by[U(A)], where w is Keynes's weight of the evidence variable, w, that measures the completeness of the relevant, available evidence upon which the probabilities p and q are calculated.(Ramsey always assumes that the value of w is 1 so that mathematical expectations can be calculated.) is an index defined on the unit interval between 0 and 1,p is the probability of success, and q is the probability of failure. p+ q sum to 1 if they are additive. This requires that w=1.Keynes's c coefficient can be rewritten as

c=p [1/(1+q)] [2w/(1+w)].

Now multiply the above by A or U(A). One obtains

cA =p[1/(1+q)] [2w/(1+w)] A or

cU(A)= p[1/(1+q)] [2w/(1+w)] U(A).

The goal is to maximize cA or cU(A). The weight 1/(1+q) deals with the linearity or non-linearity of probability preferences, which reflect situations like the certainty, reflection and translation effects. The weight 2w/(1+w) deals with non-additivity. Ramsey's theory amounts to nothing more than the claim that c=p or cA [cU(A])= pA [pU(A)] .

It is now straightforward to see that Ramsey's theory assumes that all probabilities are additive and linear. This is nothing but a special case of Keynes's generalized decision rule to maximize cA, or cU(A), as opposed to the Ramsey rule to maximize pA or pU(A). Economists today have, at best, only a very vague, hazy, or cloudy understanding of Keynes's distinction between risk and uncertainty. It is this distinction that has to be grasped first before any economist can have any hope of understanding what Keynes meant in the GT in chapter 12 on pp.161-163 of his discussions of reasonable and unreasonable calculation.

Set w=1 in c and assume that all probability preferences are linear. Both of Keynes's weights drop out and you are left with Ramsey's model. Then

Keynes's Max cA or cU(a) simplifies to Ramsey's Max pA or pU(A). Whichever demonstration the reader chooses from the above three demonstrations, the same result will be obtained. Ramsey did not know or understand anything about what Keynes was doing in the TP because he was an advocate of precise probability while Keynes was an advocate of imprecise probability.

7. Conclusion

All of Ramsey's work on Keynes from 1922 through 1926 is a God awful, bloody mess based on immense confusions that Ramsey concocted about Keynes's objective, logical relations being identical to the Platonic, metaphysical relations that underlie G E Moore's intuitionist approach in ethics. This includes all his Apostle papers mentioning Keynes or Russell's use of a propositional logic, which are actually based on Boole's 1854 work, as well as all of his personal correspondence with Keynes.

Kyburg is able to base his critique of Ramsey's attacks on Keynes on his correct assessment of Keynes's initial, preliminary, graphical exposition, as contained in a diagram on page 39, which is supported by Keynes's analysis on pp.38-40, which shows what is called a poset (partially ordered set). Kyburg clearly shows that Keynes's diagram is a mathematical structure called a lattice with a meet and join clearly defined by the existence of a greatest lower bound (glb) and a least upper bound (lub). However, Keynes also made it quite clear that this analysis was not mathematically detailed and that a detailed mathematical analysis would take place in Part II, which, as Borel realized, was the heart of Keynes's book.

The fact that no philosopher or economist or historian or social scientist or behavioral scientist or decision theorist was ever able to read Part II of the TP explains why, in order to fill the gaping hole about Keynes's logic of uncertainty, a hole recognized by Hishiyama, Moore's Platonic, metaphysical, relations and intuitionism served as the replacement for Boole's objective, logical relations which hold between all sets of RELATED propositions, either as premises or conclusions.

Kyburg thus both succeeded and failed at the same time. He succeeded in mastering Keynes's graphical analysis in Part I of the TP, but failed to grasp Keynes's advanced, technical, mathematical analysis in Part II of the TP. However, Kyburg's results are superior to the results of any other contemporary economist or philosopher writing on Keynes in the period 1960-2010.

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